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VIBE-Walk and VIBE-R: Implicit asset market yields

Extracting better price information and predicting asset price bubbles?

Working paper af Morten Vibe-Pedersen

Vi forbereder erhvervsliv og uddannelser til fremtiden gennem praksisnære forsknings- og udviklingsaktiviteter

VIBE-Walk and VIBE-R: Implicit asset market yields

Extracting better price information and predicting asset price bubbles?

Morten V.-Pedersen, 2024

Erhvervsakademi Aarhus engagerer sig i anvendt forskning, udvikling og innovation, der gavner både uddannelser, virksomheder og samfundet som helhed. Vores projekter adresserer aktuelle problemstillinger og udføres i tæt samarbejde med både studerende og virksomheder. Resultaterne af vores forskning bidrager til at styrke videngrundlaget for vores uddannelser, kan implementeres i praksis og medvirker til at udvikle og fremtidssikre vækstlaget i danske virksomheder. Dette working paper er en del af forskningsprojektet "Forecasting i SMV".

Abstract

*Variable Implicit Bond Equivalent Rates (VIBE-R) is a new simple algorithm suggested to stationarize stochastic time series with an exponential trend. VIBE-R provides a contra-cyclical predictor of medium to long term future return as well as a contra-cyclical predictor of risk. In this working paper, the algebra of the algorithm is presented, and the algorithm is demonstrated and tested to forecast medium to long term return on Danish stock index data (OMXC PI) and Shillers American SP500 stock index data. Using Shillers total return real SP500 data, the VIBE-R is shown to perform significantly better than Shillers CAPE within sample as a medium to long term stock return predictor (p-value 0,00001***), however, insignificantly better out of sample. Additionally, the method is applied to Danish nominal stock index data, which predicts low 3 year forward return and high risk at the present price level. The method applied on real SP500 data predicts a low return 10 years forward and high risk, indicating a stock market bubble at the present price levels. However, the method applied on total return real SP500 data, displaying better stationarity properties, only predicts moderately lower forward return and moderately higher risk than historical average. The method may be applied in other asset classes and may have implications for better asset allocation and long-term risk management.*

Introduction

Stock market indices are stochastic time series with an exponential trend. Normal procedure is to take the natural logarithm (Ln) to the price to linearize the data, and the first order difference (ln-returns) to achieve stationarity in the data. The best model to explain the resulting time series of ln-returns are often described as a random walk with drift, assuming efficient markets (Wooldridge 2020).

0) $\text{Ln}(\frac{P_t}{P_{t-1}}) = \mu + e_t$ The random walk (RW) model

Where μ is a constant drift term, and e_t is any weakly dependent stochastic process. (Wooldridge). The model is consistent with a constant log-linear time trend, and the model is also known as a geometric random walk, often described as the default model for stock market data. It is also the foundation for the Black- Scholes option universe.(Hull 2006)

The model indicates that the best prediction of future price is the present price added a constant drift term. The risk is measured by the volatility (standard deviation) of the ln-returns.

However, there are some unfortunate consequences applying this model. Using historical data to estimate drift and volatility, the model can lead to pro-cyclical conclusions and behavior: During a so-called bull market period, the volatility is typically low and the historical returns are high (Wooldridge 2020) indicating high future returns and low future risk, which is also the case just before a bubble burst. Conversely, during bear market periods, volatility is typically high, and the historical returns low, indicating low future returns and high risk just before the next bull market period begins.

Pro-cyclical behavior often results in destabilizing speculation and inefficient asset allocation. Investors as well as society in general would gain from contra-cyclical behavior supporting stabilizing speculation and efficient asset allocation.

In the bond market, the problem is not the same. Yield to maturity (YTM) or 0-coupon yield is contra-cyclical by nature: During a bull market, yields are falling, shoving lower future nominal return. As a risk measure in the bond market, duration is also contra cyclical and increases during a bull market for bonds. During a bear market, yield rises and duration falls, indicating higher future nominal return and lower risk forward (Hull 2006).

In the finance literature, stock prices are often described as a perpetuity with growth due to earnings plow-back, e.g. Gordons constant growth dividend model (Brealey, Myers, og Marcus 2023). Several attempts to make contra-cyclical stock price indicators has been suggested, e.g. Shillers CAPE (Shiller 2016), or the fed-funds model Engsted 1998).

This paper assumes stock market indices can be described as a perpetuity bond with re-invested coupon. An implicit stock market 0-coupon yield (VIBE-R) and duration is derived directly from the stock market time series. VIBE-R is presented as an alternative simple non-linear way to stationarize stochastic time series with an exponential time trend. In this paper, the method is applied to predict medium to long term stock returns with interesting results.

To the authors knowledge, there are no studies that develop or apply this method. Therefore, the literature cited in this study are only representing the foundation in finance and econometrics needed for the analysis.

Method

First deriving the VIBE-R algorithm, discrete notation:

Perpetuity:

K_t : Value at time t C: Constant coupon R_t : Market yield at time t (R_t>0)

1) $K_t = \frac{C}{R_t}$
2) $K_{t-1} = \frac{C}{R_{t-1}}$

B_t: Stock of perpetuity at time t with re-invested coupon:

3) $B_t = B_{t-1} + B_{t-1} * \frac{C}{K_{t-1}}$

(2) insert in (3)

4) $B_t = B_{t-1} + B_{t-1} * (C / \frac{C}{R_{t-1}})$ can be re-written

5) $B_t = B_{t-1} * (1 + R_{t-1})$ or

6) $\frac{B_t}{B_{t-1}} = (1 + R_{t-1})$

P_t: value at time t of a perpetuity with reinvested coupon:

7) $P_t = B_t * K_t$ (1) insert in (7)
8) $P_t = B_t * \frac{C}{R_t}$

Furthermore we have:

9) $P_{t-1} = B_{t-1} * K_{t-1}$ (2) insert in (9)
10) $P_{t-1} = B_{t-1} * \frac{C}{R_{t-1}}$

Divide (8) with (10) (multiplying with invers fraction):

11) $\frac{P_t}{P_{t-1}} = \frac{B_t}{B_{t-1}} * \frac{C}{R_t} * \frac{R_{t-1}}{C}$ (6) insert in (11)

12) $\frac{P_t}{P_{t-1}}$ = (1+R_{t-1}) * $\frac{R_{t-1}}{R_t}$

Or in continuous form, where Ln is the natural logarithm:

13) Ln($\boldsymbol{\frac{P_t}{P_{t-1}}}$) = $\mathbf{R_{t-1}}$ + Ln($\boldsymbol{\frac{R_{t-1}}{R_t}}$) (the **VIBE-walk model)**

A simpler and perhaps more elegant proof is also available.

The formular is easy to interpret and use. Log returns Ln($\frac{P_t}{P_{t-1}}$) is caused either by R_{t-1} or by changes in $\frac{R_{t-1}}{R_t}$. Log returns above R_{t-1} result in lower R_t,, and log returns below R_{t-1} result in higher R_t just like the yield to maturity or 0-coupon yields behaves in the bond market.

Comparing the geometric random walk (GRW) in equation 0) with 13), the constant drift rate $\boldsymbol{\mu}$ in the GRW model is in the VIBE-walk model a variable and contra cyclical drift rate $\mathbf{R_{t-1}}$, which is like the bond markets yield to maturity or 0-coupon yield.

Additionally, when rearranging equation 13), we get the algorithm for calibrating VIBE-R_t (also just denoted R_t in the rest of the paper):

14) $\mathbf{R_t}$ = EXP[Ln($\mathbf{R_{t-1}}$)+$\mathbf{R_{t-1}}$- Ln($\boldsymbol{\frac{P_t}{P_{t-1}}}$)] where EXP is the exponential function.

Now, the R_t time series can be recursively calculated directly from the P_t time series using an appropriate start parameter R_0. Remark, that an infinite number of different R_t series can be generated using different start parameter R_0.

R_0 is the only, but very crucial parameter for the method. If R_0 is chosen to low, the R_t series will asymptotically move towards 0. If R_0 is chosen to high, the R_t series will explode. Throughout this paper, R_0 is calibrated to minimize the variance of the Ln(R_t) series.

Even small changes in the start parameter R_0 result in large changes in the "tail" of the R_t series, especially when the time series is long. Therefore, the assessment of the method as a predictor for future long-term return and risk crucially depends on out-of-sample testing, where the determination of R_0 consistently is based only on information from the with-in sample period, and consistent use of the same optimization method. (minimizing the variance of ln(R_t))

The R_t series can be used to calculate a non-linear trendline in the original series:

ln($\boldsymbol{\frac{P_t}{P_{t-1}}}$) = $\mathbf{R_{t-1}}$ + ln($\boldsymbol{\frac{R_{t-1}}{R_t}}$) can be written as

15) $\ln \boldsymbol{P_t} = \ln \boldsymbol{P_0} + \sum_{n=0}^{t-1} R_n + \ln R_0 - \ln R_t$

Taking the expected value of $\ln R_t$ and adding an error term $\mathbf{e_t}$

16) $\ln \boldsymbol{P_t} = \ln \boldsymbol{P_0} + \sum_{n=0}^{t-1} R_n + \ln R_0 - E(\ln R_t) + e_t$

We can easily calculate the nonlinear trend E($\ln P_t$) (the VIBE-path) from the R_t series:

17) $E(\ln P_t) = \ln P_0 + \sum_{n=0}^{t-1} R_n + \ln R_0 - E(\ln R_t)$ **VIBE-path**

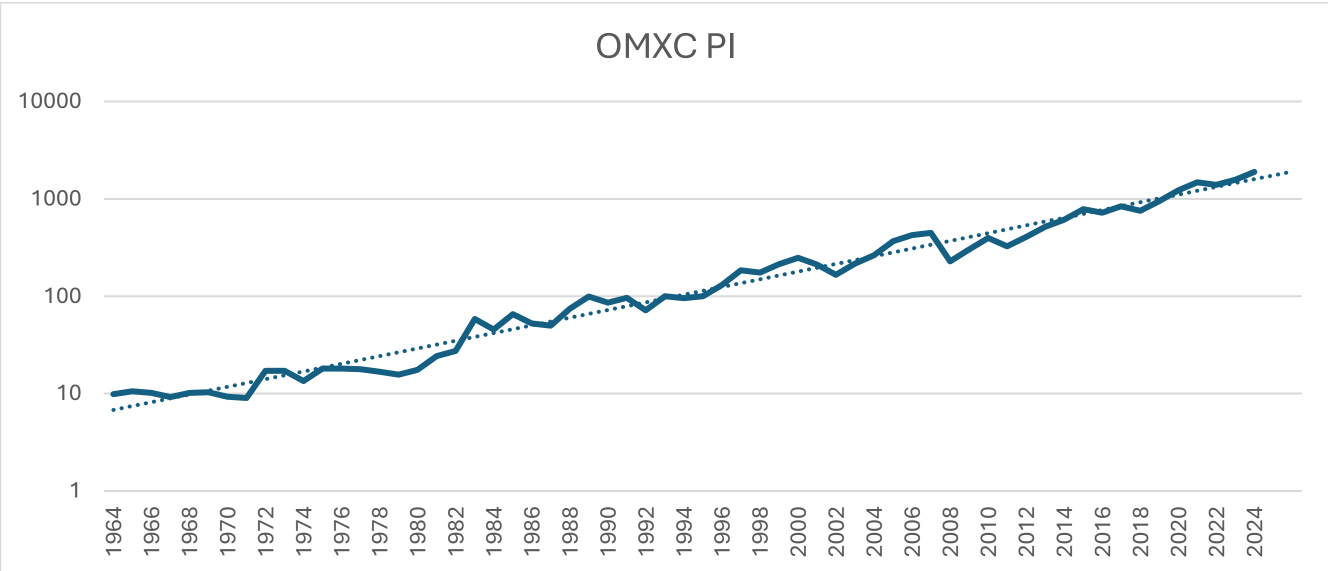
In this paper, R_t will be calibrated and tested for stationarity and prediction power within sample and out of sample on two datasets: The Danish total stock market index OMXC PI 1964-2024 (annual data, appendix 1) and SP500 data from Shiller 1871-2024 (monthly data). The Danish OMXC PI data will be used to illustrate how the methods works applying a relatively simple dataset, and the methods are then applied analyzing the much larger and more widely known SP500 dataset, provided by Shiller (Shiller 2024).

All data will be available for the reader in appendix 1 or online data (Shiller 2024). All calculations and graphics can be reproduced by the reader with an excel spreadsheet with standard functions including a solver algorithm.

Results:

Danish stock market data (OMXC PI) 1964-2024 annual data (appendix 1)

Fig. 1 OMXC PI 1964-2024 annual data, log scale



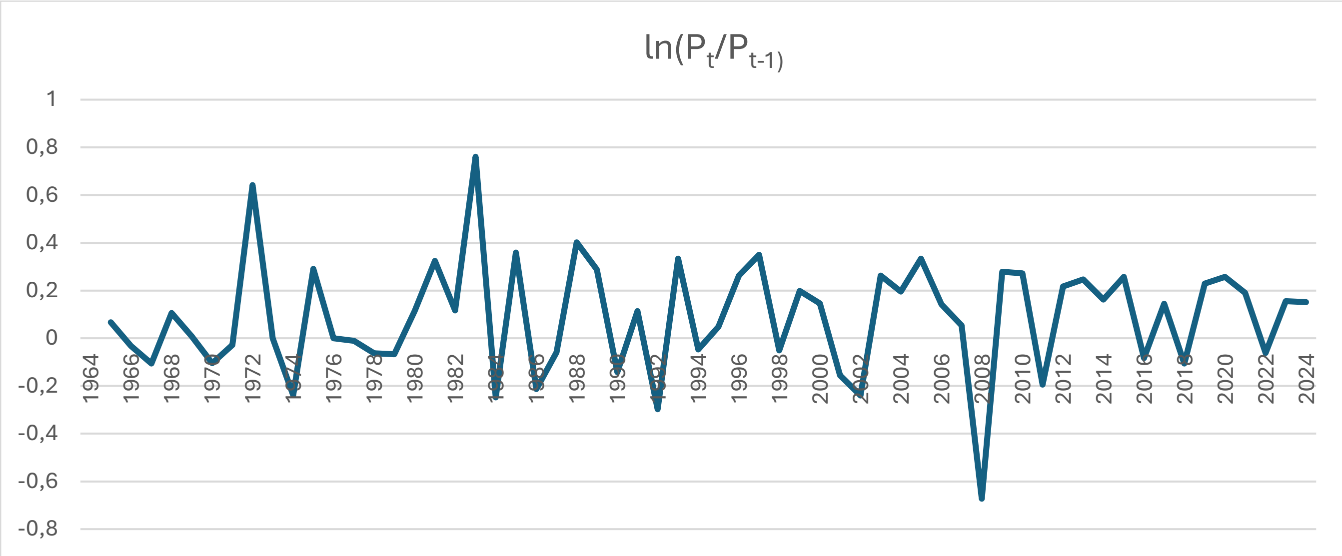
Source: Danmarks statistik, 10y reviews and statistikbanken. Excel graphics. (raw data in appendix 1) Last observation 11/03/2024, P_t =1890

Fig 1. shows the Danish total share index. It is an example of a stochastic time series with an exponential trend, and there appears to be mean reversion to a linear time trend in the sample period.

Random walk with drift or a VIBE-walk?

Taking the first order difference:

Fig . 2



Source: own calculations, excel graphics.

Dickey Fuller (DF) t-value : -9,349 H0: Unit root: Can be rejected. (Wooldridge 2020). Stationarity is clearly achieved by differentiating the $\ln P_t$ series.

Autocorrelation:

Lag	1	2	3	4	5	6	7
Autocorr	-0,211	-0,090	-0,052	-0,088	-0,136	0,114	-0,072
+/- crit.	0,25517	0,257361	0,259608	0,261916	0,264286	0,266722	0,269227

No significant autocorrelation suggesting **$\mathrm{Ln}(\frac{P_t}{P_{t-1}})$** is an ARIMA(0,0,0) process. Nothing rejects the random walk hypothesis:

Recall 0) $\mathrm{Ln}(\frac{P_t}{P_{t-1}}) = \mu + e_t$

As expected, there is no predictive power in the $\ln(P_t/P_{t-1})$series. Since the price level is differentiated away, no mean reversion to the exponential trend can be detected using this model. Using traditional log-return to stationarize the P_t series, the only predictive power is the constant drift term μ.

Applying the VIBE-walk, calculating the VIBE-R series:

Recall (13) $\mathrm{Ln}(\frac{P_t}{P_{t-1}}) = R_{t-1} + \mathrm{Ln}(\frac{R_{t-1}}{R_t})$ can be rearranged to:

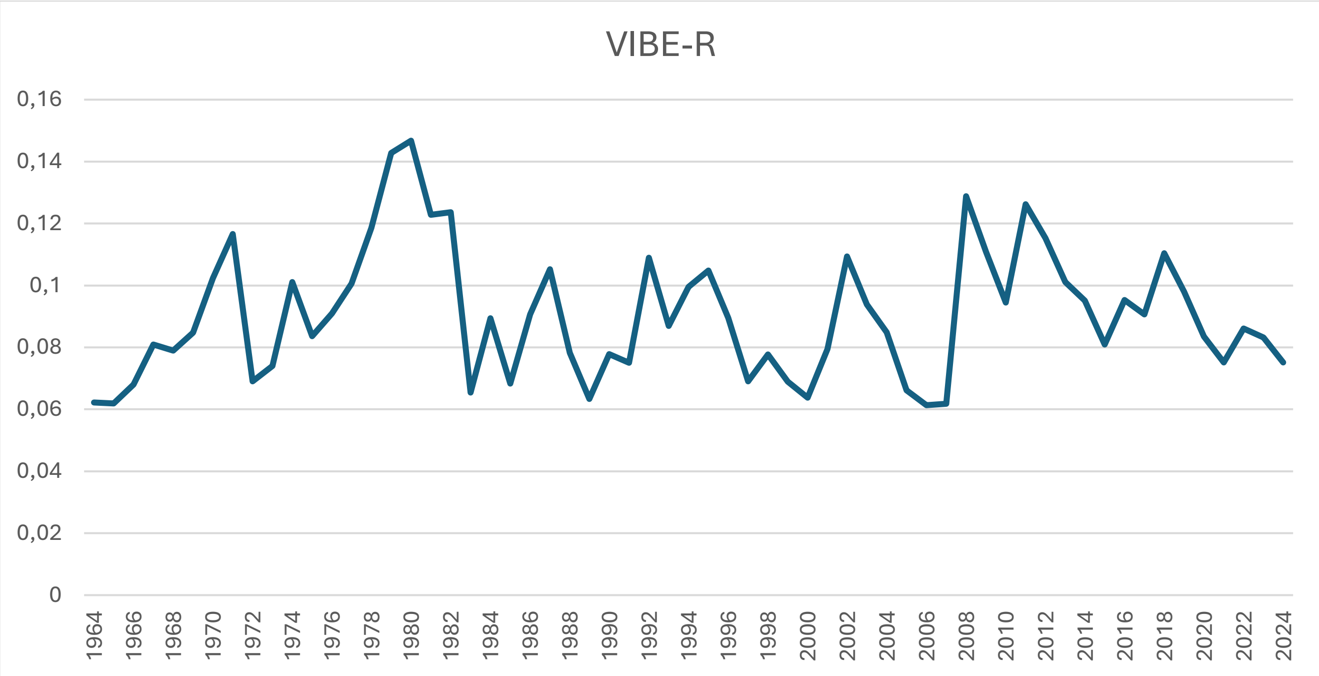
Recall (14) $R_t = \mathrm{EXP}[\, \mathrm{Ln}(R_{t-1}) + R_{t-1} - \mathrm{Ln}(\frac{P_t}{P_{t-1}})\,]$

and $\mathbf{R_t}$ can be recursively calculated from a start parameter R_0.

Recall that if the R_t series will explode if a to high R_0 is chosen, and the R_t series will asymptotically approach 0 if a to low value of R_0 is chosen. Therefore, the optimal R_0 is estimated by minimizing the variance of the $\ln R_t$ series, which gives the best stationarity of the $\ln R_t$ series, using an iterative tool (excel solver is used though out the paper). In this case R_0 = 0,062167 minimizes the $\ln R_t$ series.

Other optimization criteria are briefly discussed in the final section of this paper.

Fig. 3



Source: own calculations and excel graphics.

Dickey – Fuller (D.F.) t-value -4,379 H0: Unit root: Can be rejected.

Stationarity in the time series is in this case achieved, and this is an important quality for the use of the method, as will be demonstrated later.

Autocorrelation (ACF) and partiel autocorrelation (PACF) calculated on $\ln R_t$:

Lag	1	2	3	4	5	6	7
ACF	0,520414	0,233177	0,044063	-0,11764	-0,15483	-0,06813	-0,02979
+/-crit	0,25517	0,257361	0,259608	0,261916	0,264286	0,266722	0,269227
PACF	0,520414	-0,0377	-0,067	-0,128	-0,026	0,067	-0,0336
+/- crit	0,25517	0,26	0,266	0,271	0,278	0,283	0,291

The ACF and PACF indicates $\ln R_t$ is an autoregressive process of order 1 (ARIMA (1,0,0)).

18) $\ln R_t = -1{,}2295 + 0{,}4917 * \ln R_{t-1} + e_t$

S.E. (0,272) (0,112)

Testing for autocorrelation in the error term e_t: (source):

16) $e_t = 0,0031 + 0,00039*e_{t-1}$

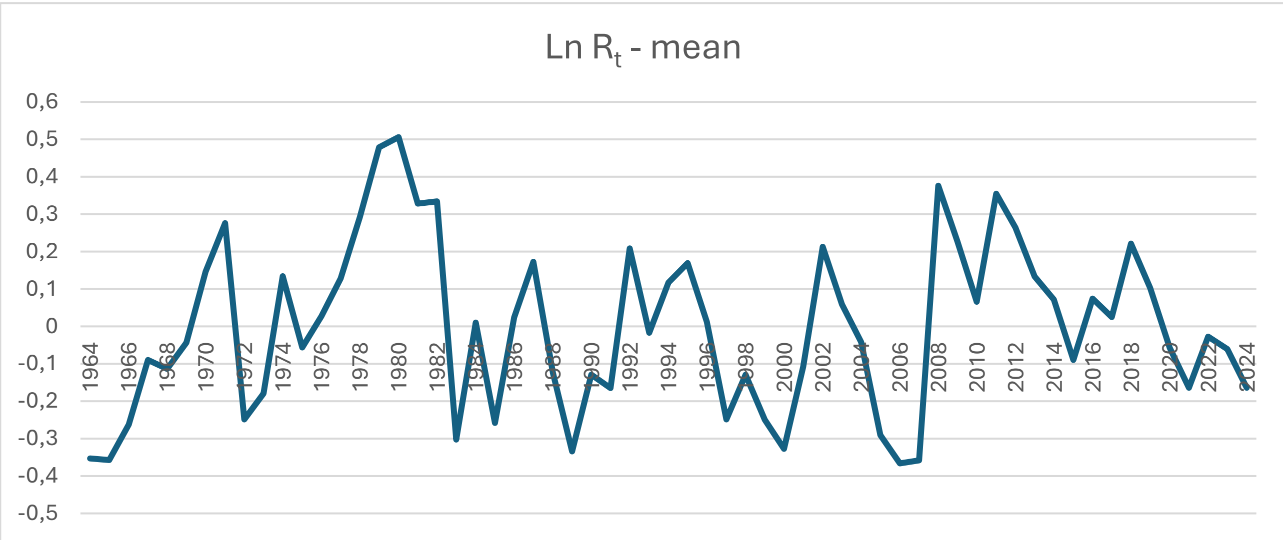
S.E. (0,025) (0,132)

No autocorrelation is detected in the error term. Durbin Watson (DW) test value: 1,97

The R_t is by nature an implicit perpetuity bond yield, varying inversely with the stock price. Expected future return high after bear markets in 1980, 1992, 2002, 2008, 2011 and low expected future return after bull markets in 1965, 1972, 1983, 1989, 2000, 2006 – at present (marts 2024) expected future return is below average.

The level-variation of the original time series P_t is maintained, so clear mean reversion to the exponential trend in the original series can be detected as seen in the AR1 model equation 15). The lnR_t series is "mirroring" inversely the deviations from the log-linear trend in fig. 1. Taking the natural logarithm to the R_t series and subtracting the mean gives the excess expected future return above the average expected future return:

Fig.4 4

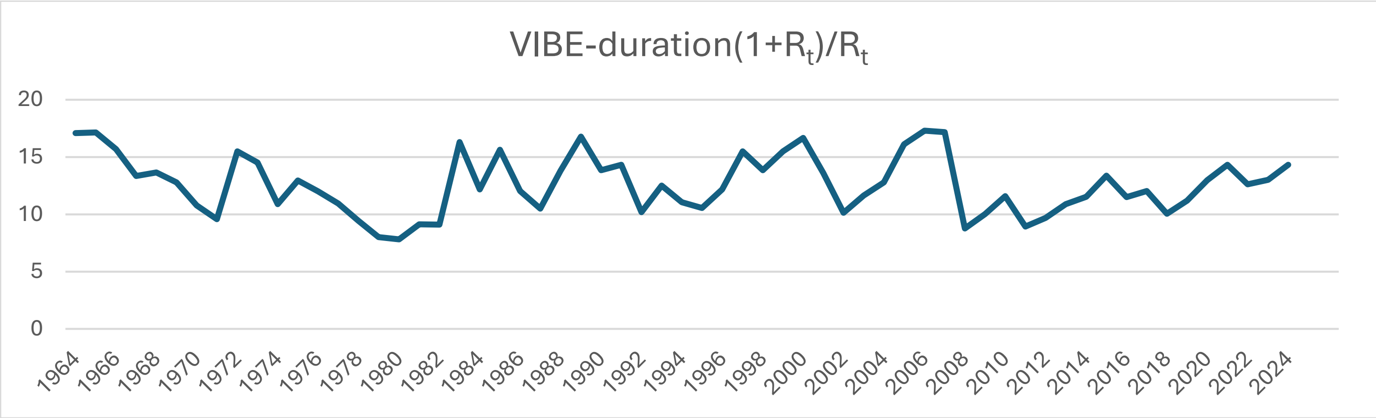


Source: Own calculations and excel graphics

The VIBE-duration

Duration on an perpetuity bond is $(1+R_t)/R_t$ (Hull 2006). Calculating the implicit duration using VIBE-R in fig. 3 result in fig 5:

Fig. 5

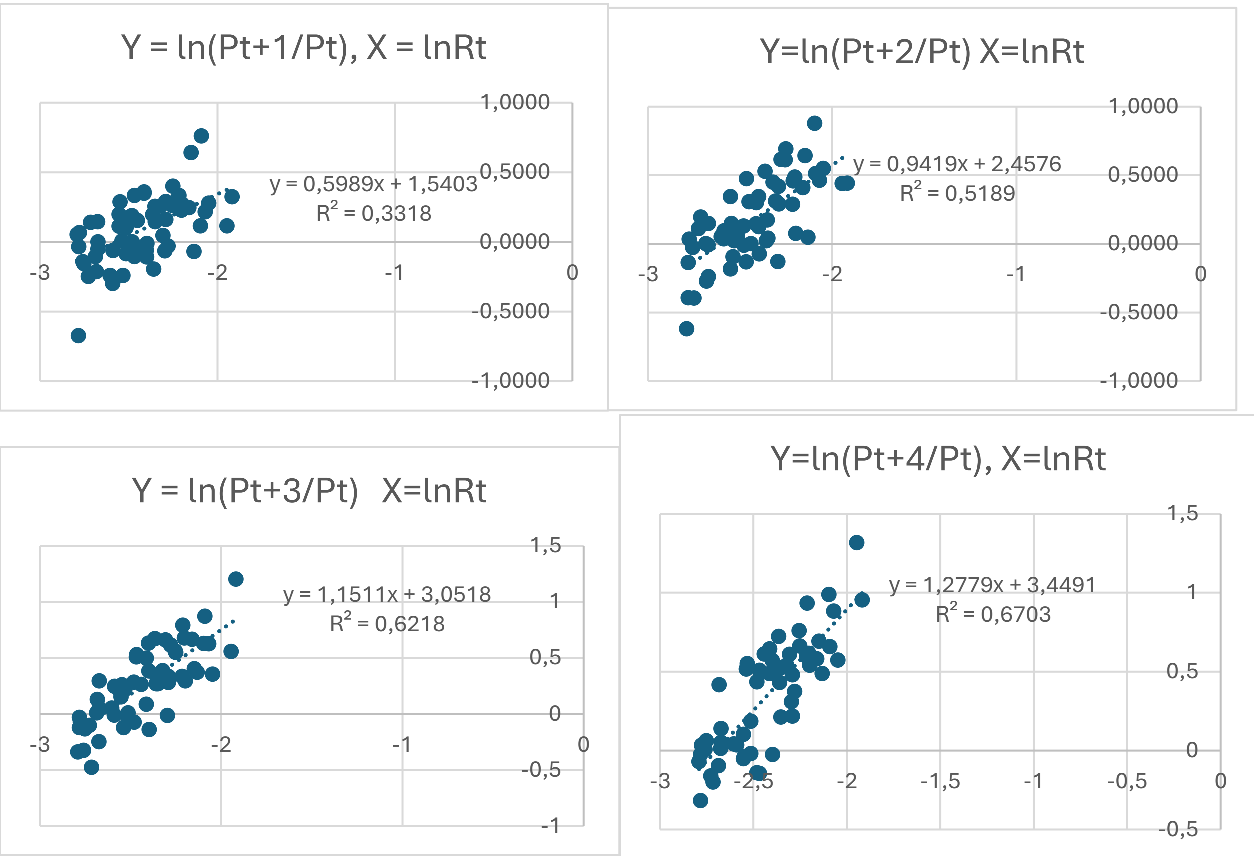


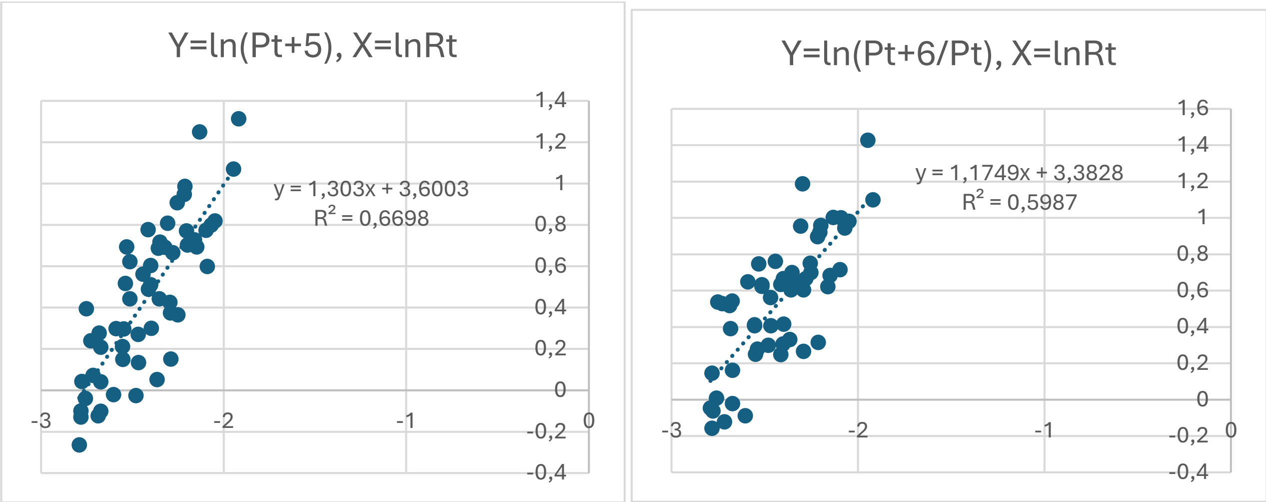
The VIBE-duration varies inversely with VIBE-R_t showing low risk after bear markets in 1980, 1992, 2002, 2008, 2011 and high risk after bull markets in 1965, 1972, 1983, 1989, 2000, 2006 – at present (marts 2024) risk is above average.

An alternative interpretation of the VIBE-duration is to view it as a kind of implicit Price/Earnings ratio. Later in the paper the VIBE-duration is compared to Shillers CAPE on SP500 stock market data.

VIBE-R power of prediction, within sample: Scatterplot of $\ln R_t$ and $\ln(P_{t+n}/P_t)$:

Fig.6





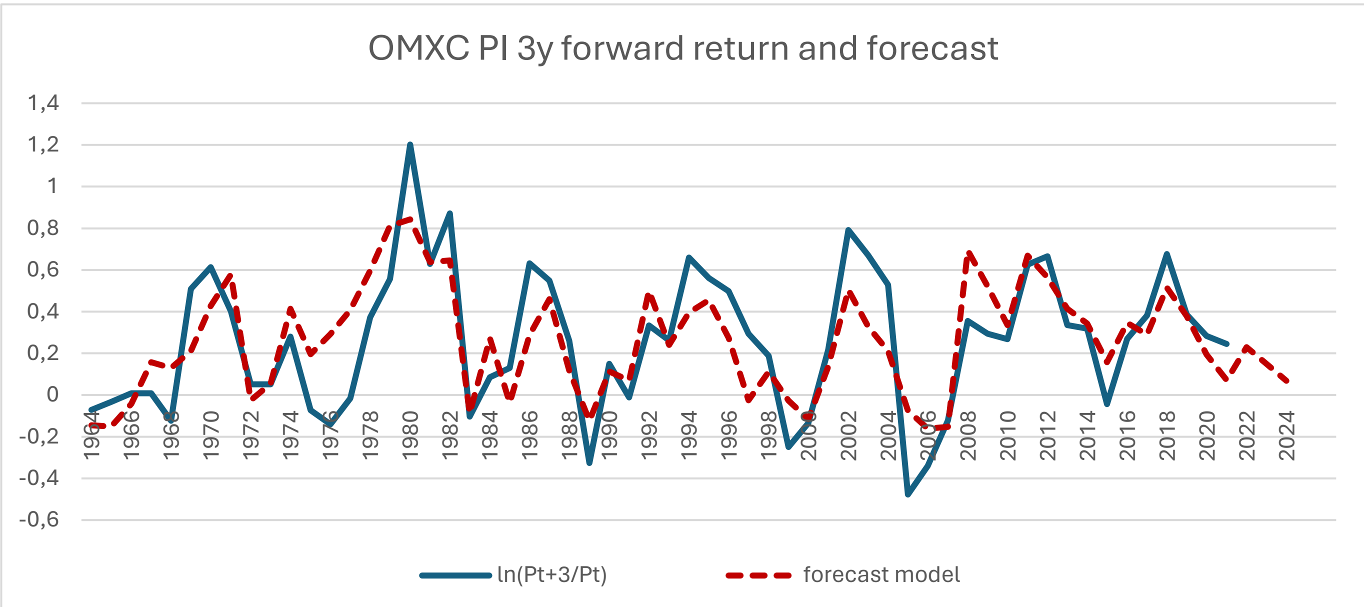
Source: Own calculations and excel graphics

Fig. 6 shows that the predictive power measured as R^2 is highest 3-4 years forward, indicating the length of mean reversion time – corresponding with the estimated AR1 model in equation 18).

As we shall see later using SP500 index data, mean reversion is slower on US stocks.

A VIBE-R-prediction model for 3 y forward return on OMXC PI in a time plot:

Fig 7:



Source: Own calculations and excel graphics

The prediction power within sample seems impressive, but it is very important to notice, that the predictions out of sample is very sensitive to small changes in the crucial start parameter R_0.

Out of sample test are therefore essential for evaluating the whole method. Provided the time series is long, the results are quite robust and stable, due to the stationarity of the VIBE-R series in this case. The out of sample prediction show lower than average forward return and higher than average risk at the present price (11/3 2024 P_t = 1890)

Going back in the data history, and testing out-of sample shows quite stable results:

Out of sample test (OOS-test) 2011 – predicting bull market 3 years forward: R_0 = 0,062125 minimizing Variance($\ln R_t$) using excel solver,

Fig. 8

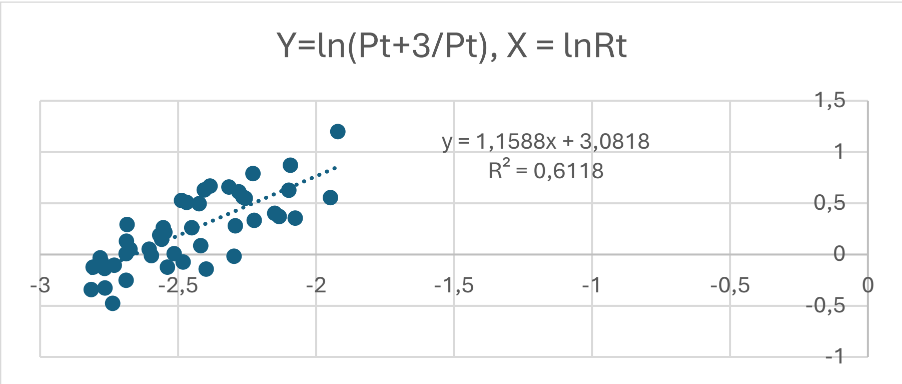
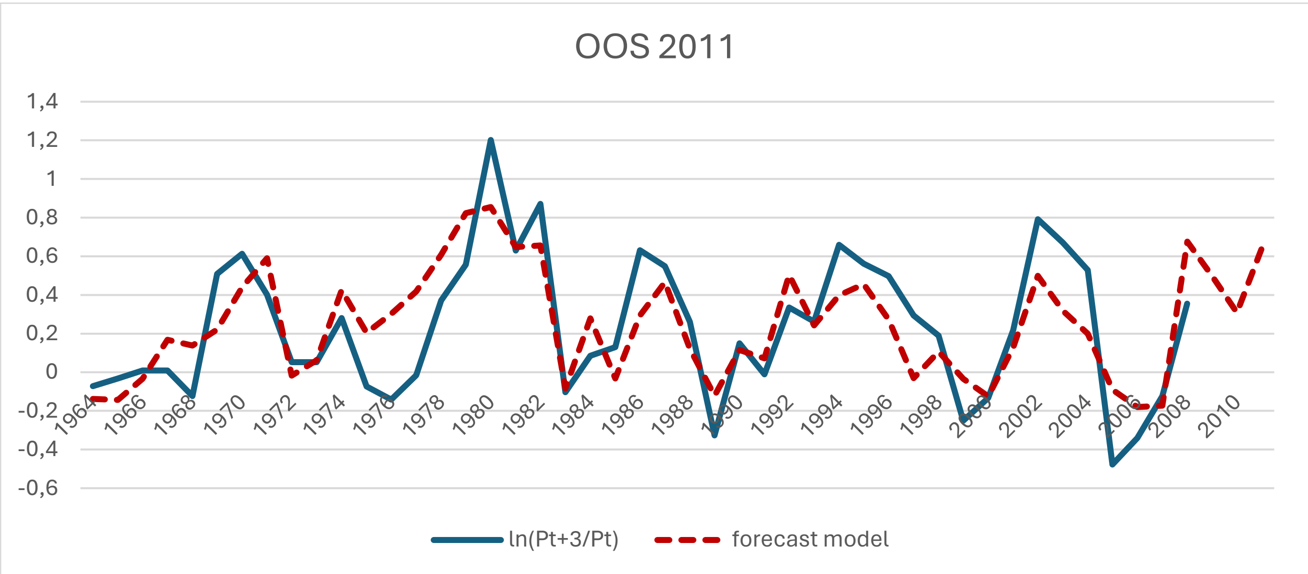


Fig. 9

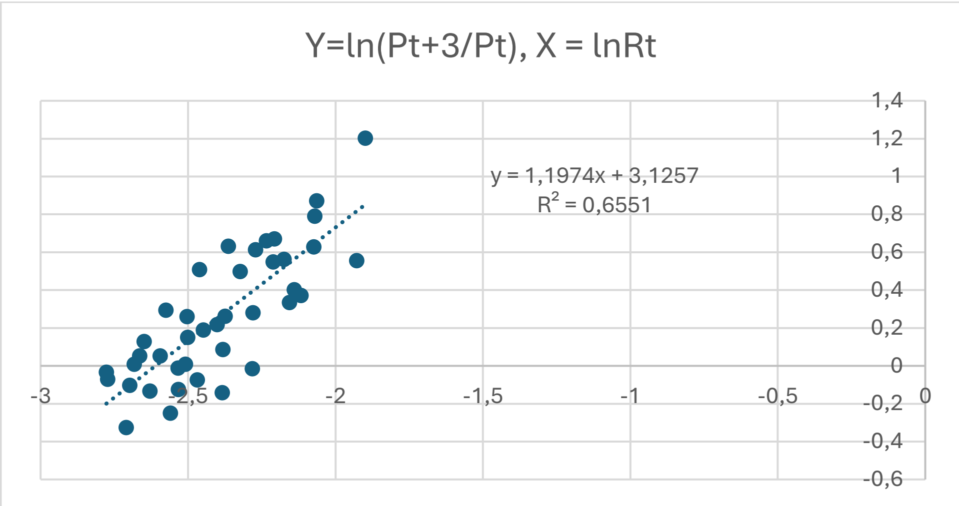


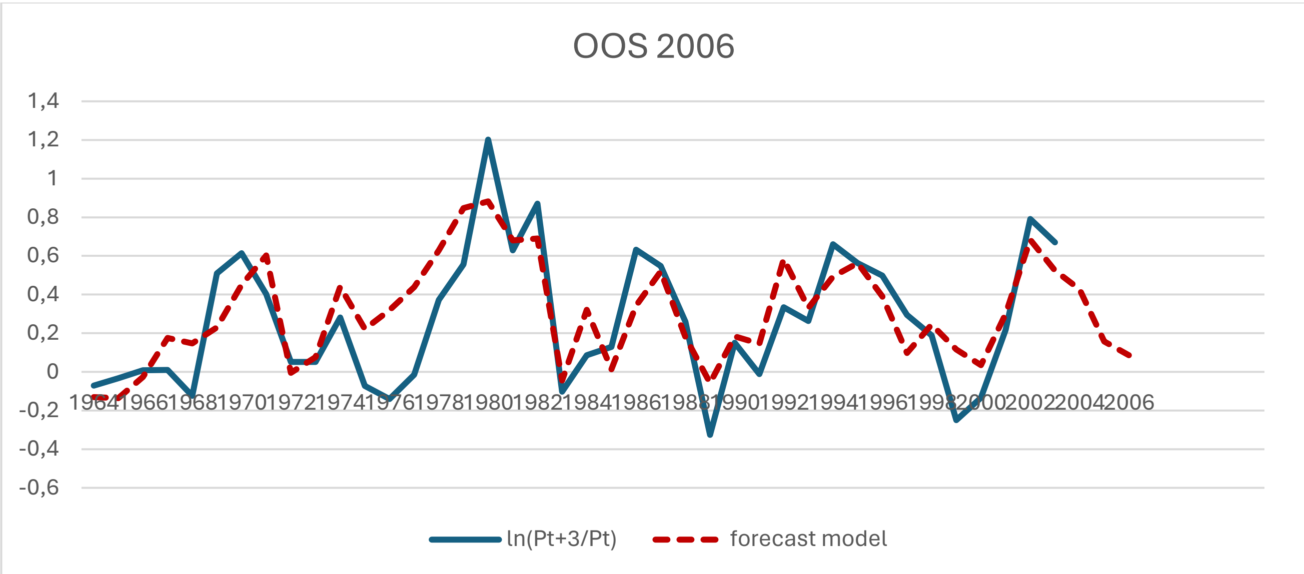
Source: Own calculations and excel graphics

Applying the method in 2011 after the “double dip” in the economy, following the financial crises of 2007-2008 forecasted very high 3 year forward return and very low risk.

Predicting bear market before the financial crises: Out of sample test (OOS-test) 2006: R_0 = 0,0624935   minimizing Variance($\ln R_t$) using excel solver

Fig. 10





Source: Own calculations and excel graphics

The method applied in 2006 before the financial crises erupted, predicted very low returns and high risk.

The method applied on Danish annual nominal stock market data 1964-2024 appears to produce robust contracyclical forecast on 3 year forward return and risk out-of sample.

VIBE-path – a nonlineær trendline in the original P_t dataset:

Recall, that the VIBE-R series can be used to generate a non-linear trendline in the original P_t-series:

(13) $\ln(\frac{P_t}{P_{t-1}}) = R_{t-1} + \ln(\frac{R_{t-1}}{R_t})$ can be written as

(15) $\ln P_t = \ln P_0 + \sum_{n=0}^{t-1} R_n + \ln R_0 - \ln R_t$

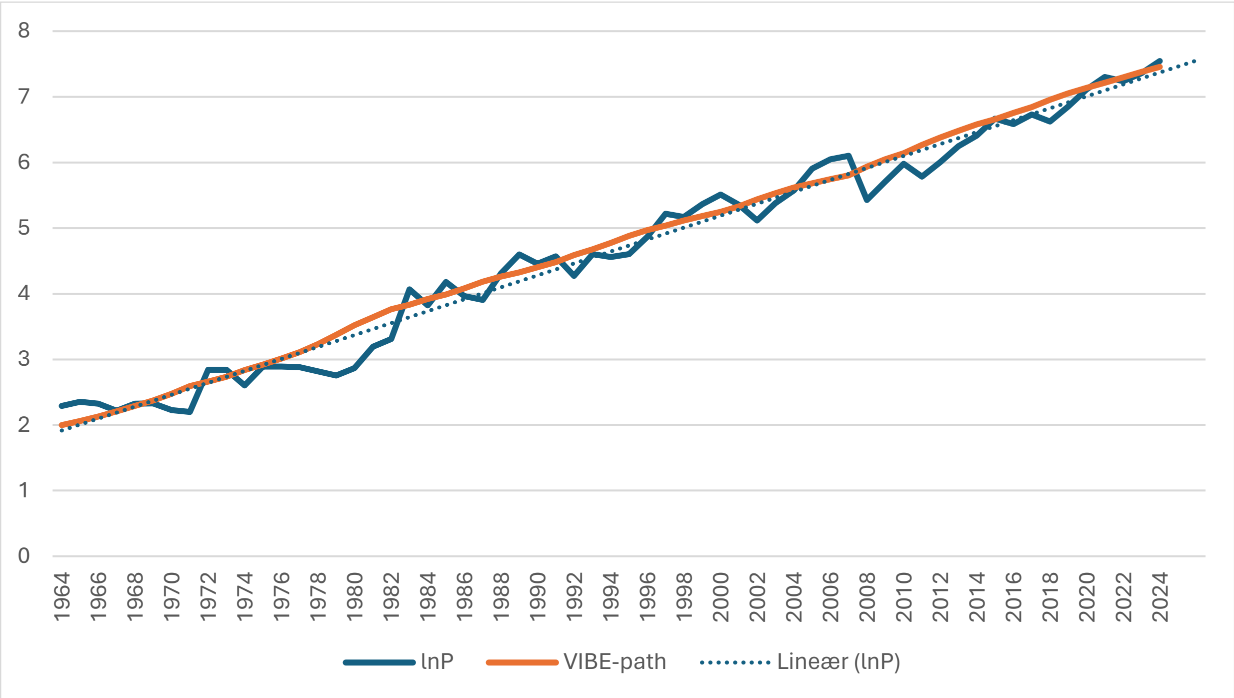
Taking the expected value of $\ln R_t$ and adding an error term $\mathbf{e_t}$

(16) $\ln P_t = \ln P_0 + \sum_{n=0}^{t-1} R_n + \ln R_0 - E(\ln R_t) + e_t$

We can easily calculate the nonlinear trend $E(\ln P_t)$ (VIBE-path) in the original dataseries from the R_t series:

(17) VIBE-path: $E(\ln P_t) = \ln P_0 + \sum_{n=0}^{t-1} R_n + \ln R_0 - E(\ln R_t)$

Fig 11



Source: Own calculations and excel graphics

Linear time trend: lnP = 0,0909t+1,8249+ e (t=1..61) SSE: sum(e^2) = 3,49

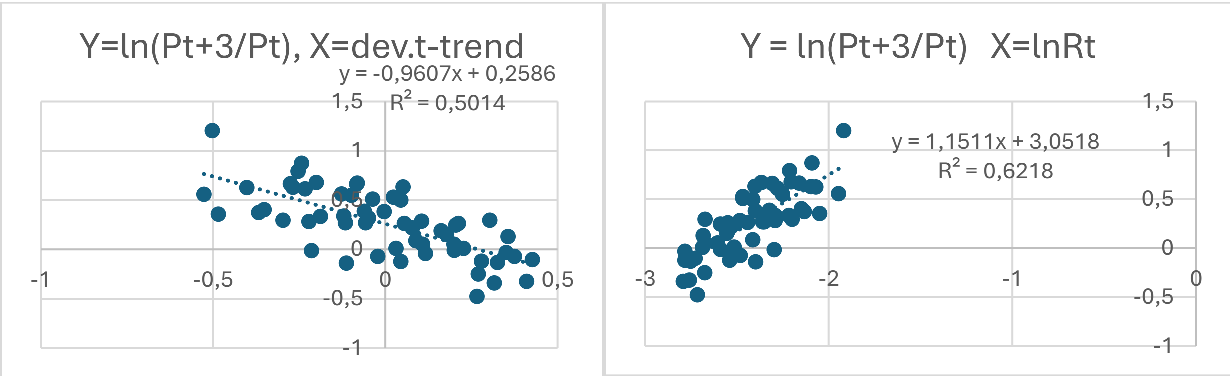
VIBE-path : $\ln P_t = \ln P_0 + \sum_{n=0}^{t-1} R_n + \ln R_0 - E(\ln R_t) + e_t$ (R_0= 0,062167) SSE: sum(e^2) = 3,01

Remark how the VIBE-path is contra-cyclical. During bull markets R_t fall, so the increase in the path slows down and vice versa in bear markets. In this Danish stock case, the non-linear VIBE-path is close to the linear time-trend, but with a slightly less SSE (insignificant).

The $\ln R_t$ -forecast model performs in this case slightly better within sample than a forecast-model based on deviations from the linear time trend:

Comparing $\ln R_t$ – forecast with time-trend-deviation model 3y:

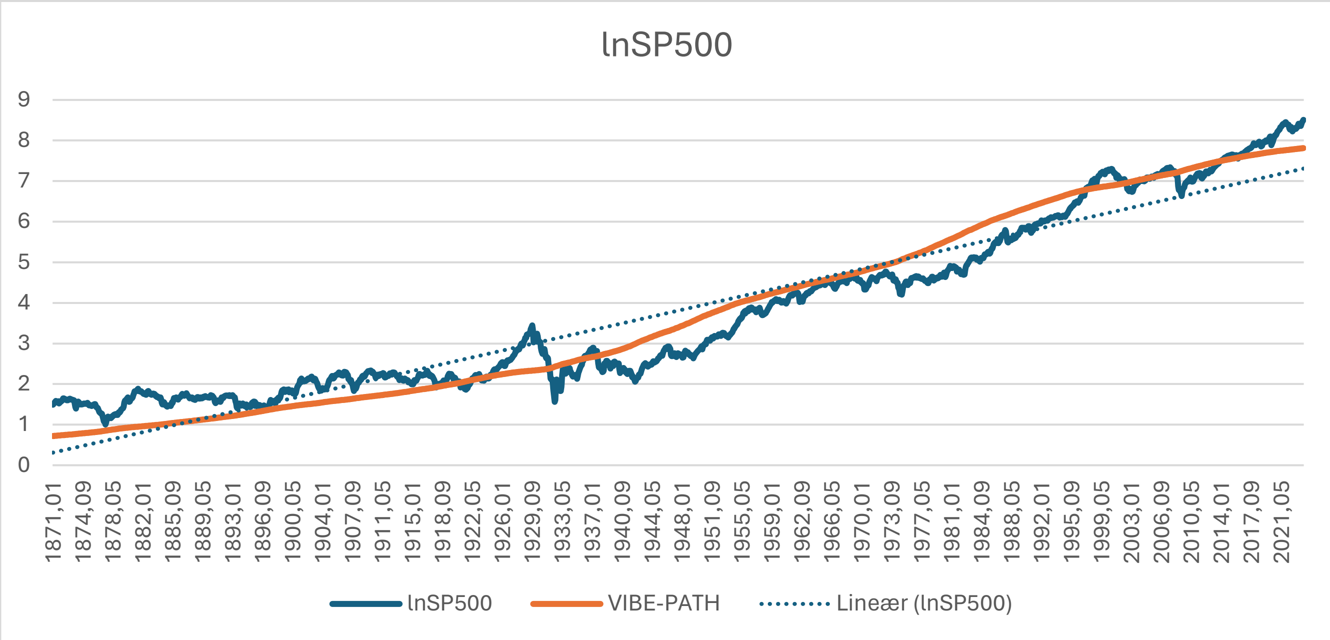
Fig 12



VIBE-R on US stock market data (SP500 – case Shiller)

Turning to a much larger and more internationally recognized dataset, we will apply the above methods on Shillers SP500 monthly data from 1871.01 to 2024.02 (Shiller 2024). Last observation is 2. February closing price P_t = 4959. The methods are just as introduced above using Danish stock market data.

Fig 13



Source: Case Shiller data (Shiller 2024), own calculations and excel graphics

Linear time trend: $\ln P_t = 0{,}0038 \cdot t + 0{,}3067 + e_t$ (t=1..N) SSE: = 770,1

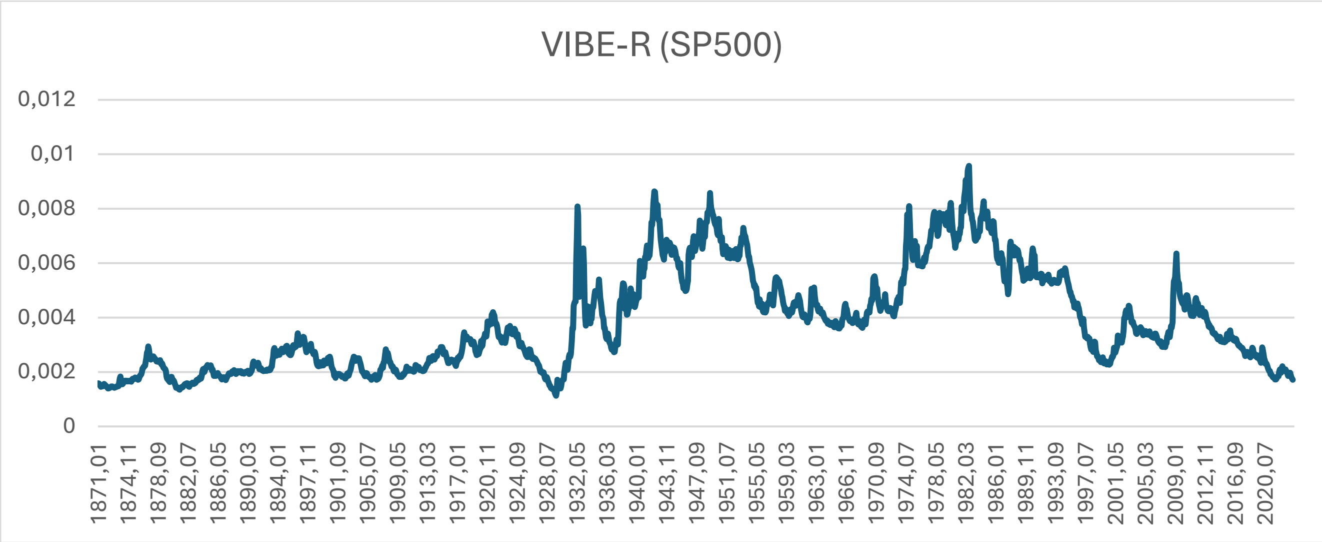
VIBE-path: $\ln P_t = \ln P_0 + \sum_{n=0}^{t-1} R_n + \ln R_0 - E(\ln R_t) + e_t$ (R_0= 0,00158894933295448) SSE = 432,2

VIBE-path shows significant lower SSE (P-value 0,000001)

This very long SP500 dataset (fig 13) do not show the same nice mean reversion to the log-linear trend as the Danish OMXC PI dataset. It also clearly shows the problem of estimating and using log-linear time trend. Using the sub-period 1871-1925 would result in a less steep line, estimating lower expected return, and using the sub-period 1945-2000 would result in a much steeper line, estimating higher expected return.

The VIBE-path is based on the VIBE-R below in fig. 14, calibrated minimizing the variance of $\ln(R_t)$: $\mathbf{R_t = EXP[\,Ln(R_{t-1}) + R_{t-1} - Ln(\frac{P_t}{P_{t-1}})\,]}$ (R_0 = 0,00158894933295448). Remark, that it is implicit monthly yields.

Fig. 14



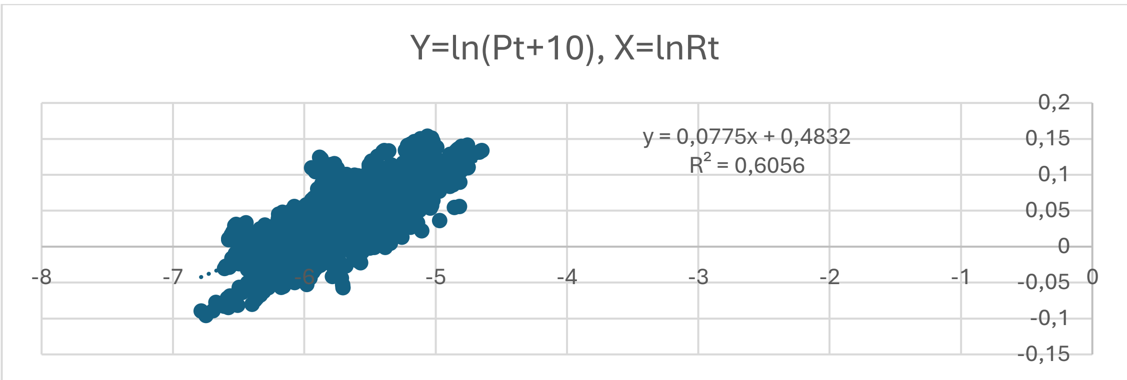
Source: Own calculations and excel graphics, based on case Shiller dataset.(Shiller 2024)

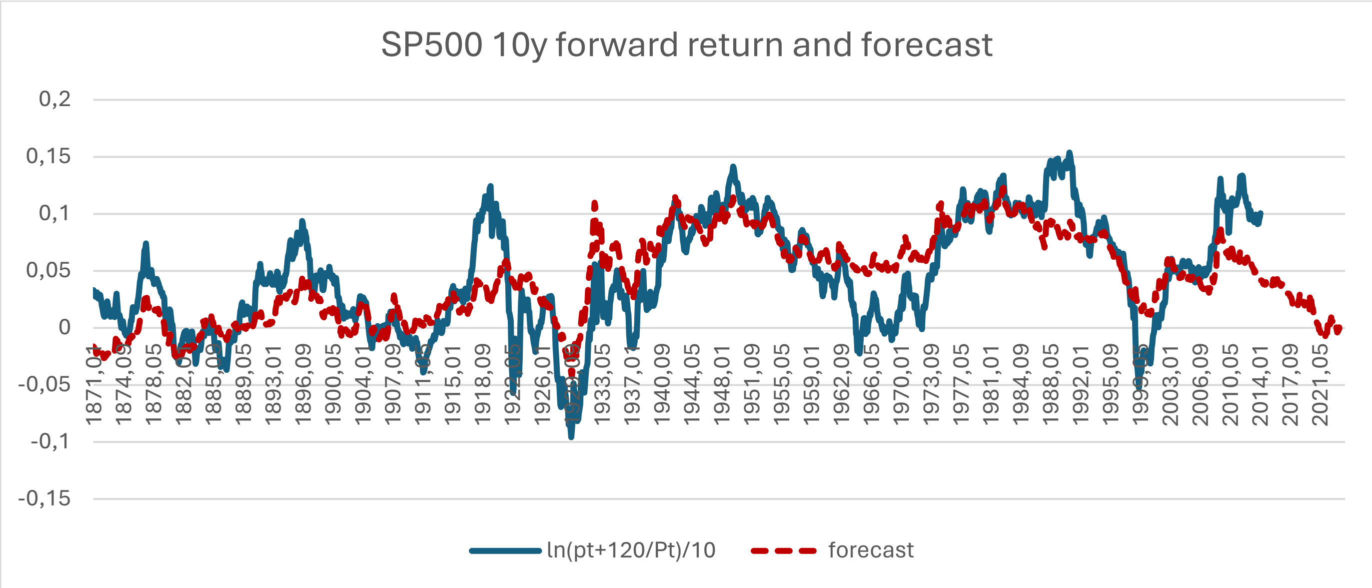
D.F. t-value: -1,86 H0: Unit root : Cannot be rejected.

This VIBE-R series is in this case not stationary. Using VIBE-R for medium to long term forecasting and risk measurement out-of-sample is now very questionable. The same problem as described above using the log-linear time-trend: Estimating VIBE-R (minimizing var($\ln R_t$)) in different sub-periods (e.g. the subperiod 1980-2024) will lead to very different results out of sample.

The mean reversion is weak and slow compared to the former case (OMXC PI). A 10 year VIBE-forecast give the following results:

Fig 15



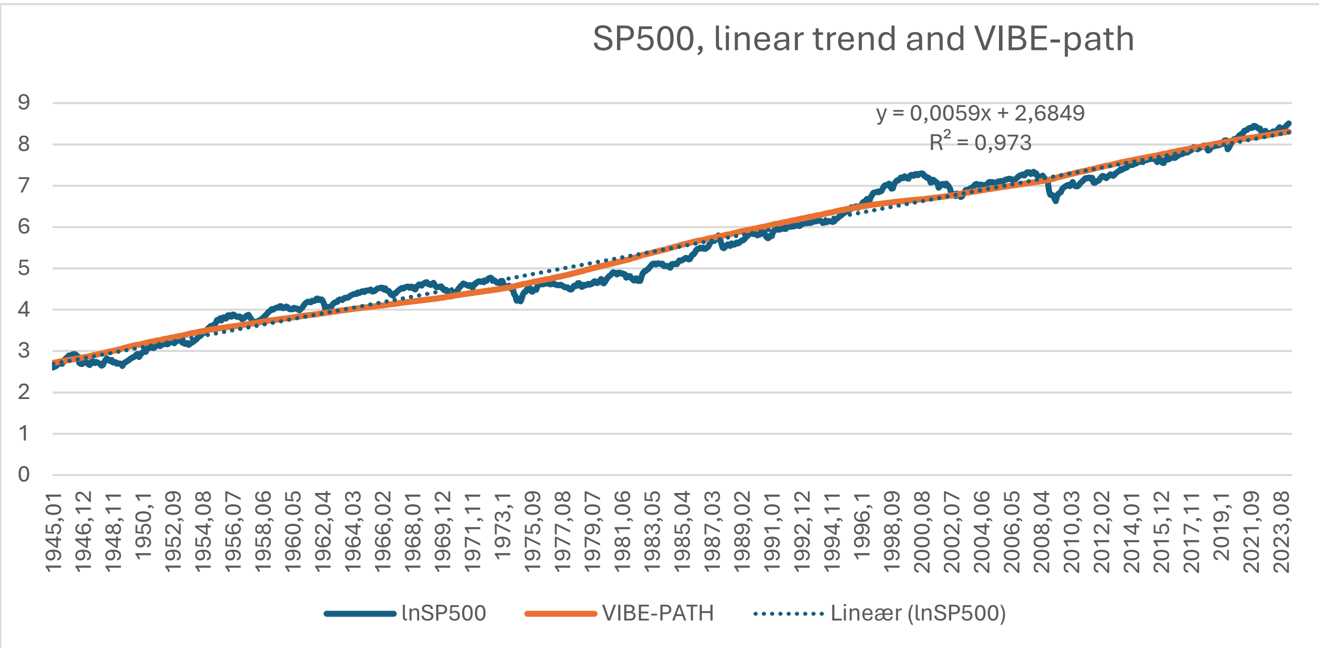


Source: Own calculations and excel graphics, based on case Shiller dataset

The last data is February 2. 2024 closing price, and the forecast model predicts a negative return over the next 10 years! Out of sample testing with the same starting point (1871.01) going back on the last data is quite robust – but the results are not stable out of sample if the starting point data is changes, due to the lack of stationarity as mentioned above. Let us study an example:

Choosing the sub-period 1945.01 to 2024.02:

Fig.16 15



Source: Own excel graphics, based on case Shiller dataset

Linear time trend: $\ln P = 0{,}0059 \cdot t + 2{,}6849 + e_t$ (t=1..N) SSE: sum(e^2) = 68,9

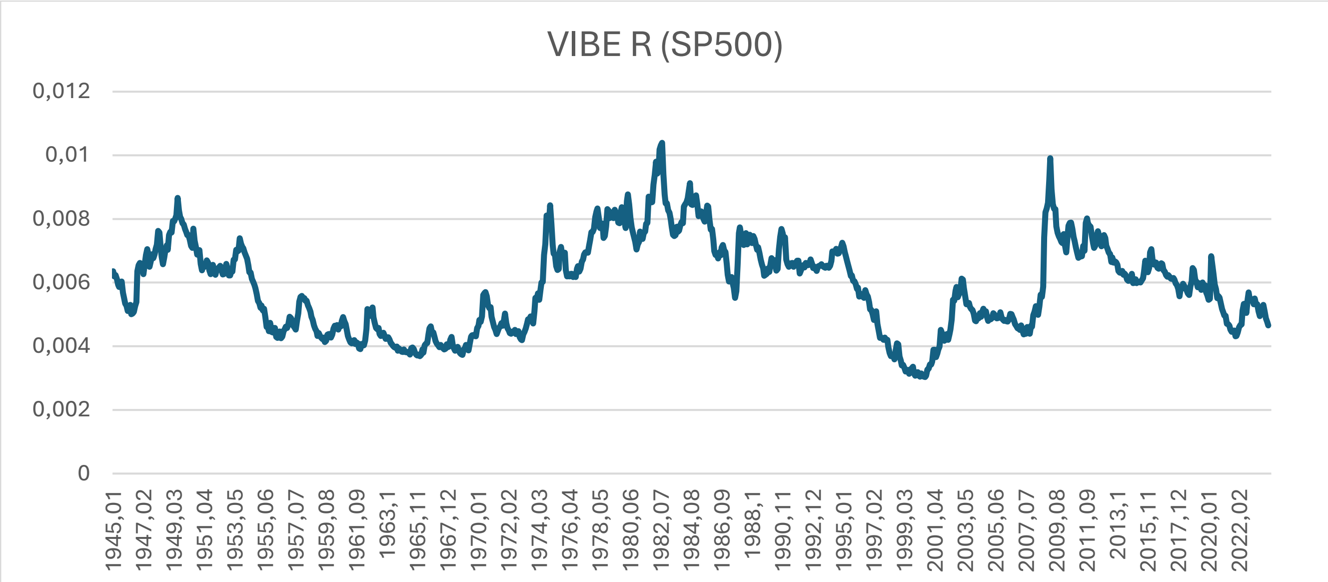
VIBE-path: $\ln P_t = \ln P_0 + \sum_{n=0}^{t-1} R_n + \ln R_0 - E(\ln R_t) + e_t$ (R_0= 0,00635771100140291)
SSE: sum(e^2) = 61,6

VIBE-path shows slightly less SSE (P-value 0,084)

This is looking like a more "pleasant" period with mean reversion to the trendline – although slow. The VIBE-path is based on the VIBE-R below in fig. 17, calibrated minimizing var($\ln R_t$):

$$\mathbf{R_t = EXP[\ Ln(R_{t-1}) + R_{t-1} - Ln(\frac{P_t}{P_{t-1}})\ ]}$$ (R_0 =0,00635771100140291)

Fig 17

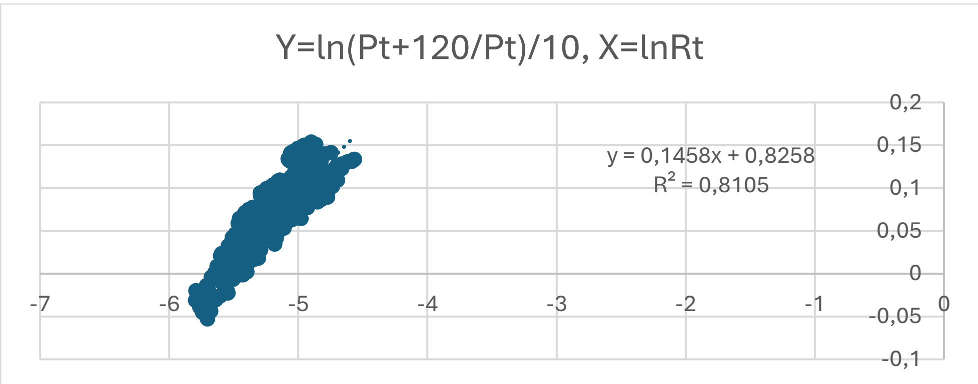


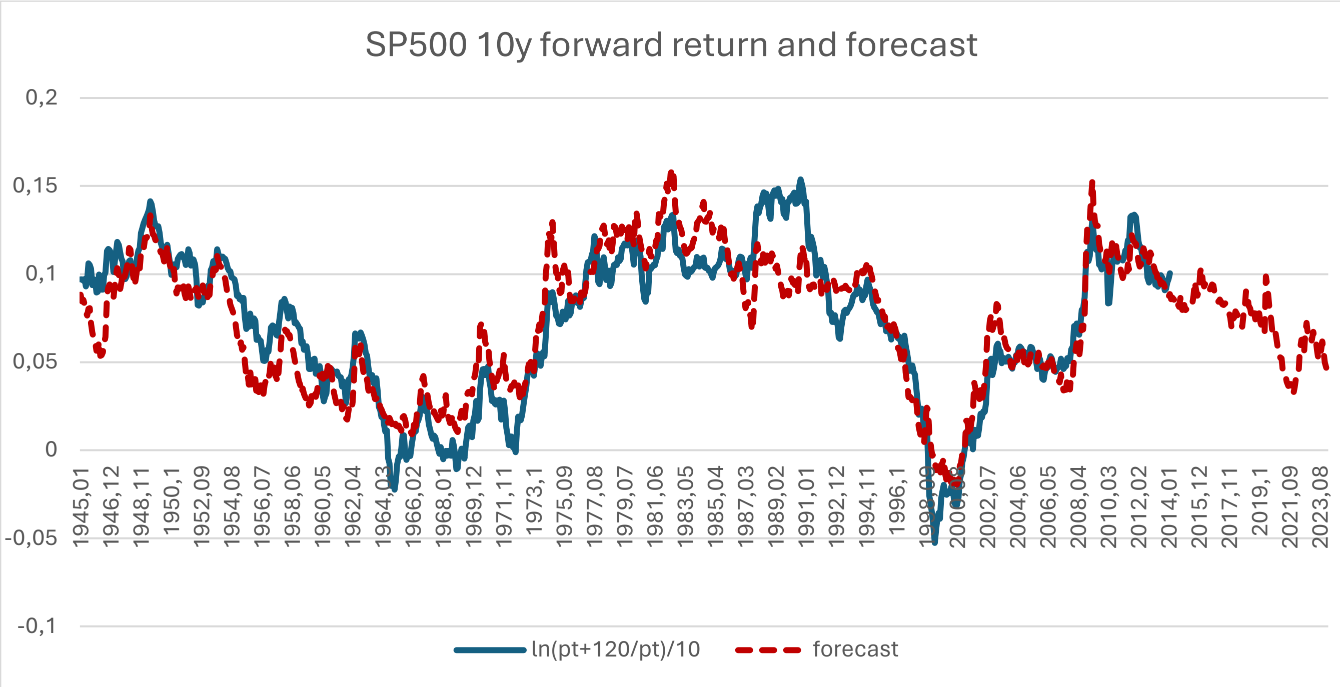
Source: Own calculations and excel graphics, based on case Shiller dataset

D.F. t-value: -2,09 H0: unit root: Cannot be rejected. Stationarity is not achieved.

10 y forecast model:

Fig 18





Source: Own calculations and excel graphics, based on case Shiller dataset

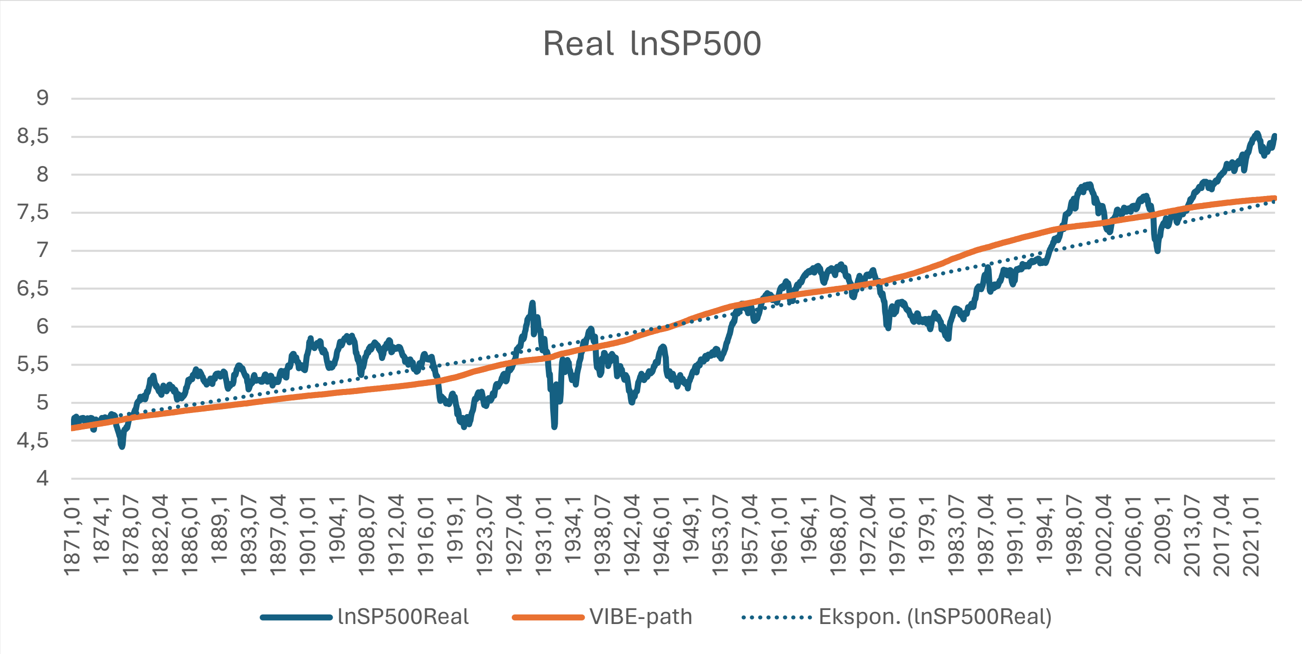
The prediction power within sample measured by R^2 is clearly higher than for the full dataset in fig. 15 above, and the out-of-sample forecast show a less dramatic prediction of a 10Y forward return (annually) a little less than average for the whole estimation period. The impressive within sample performance can be deceiving – it is the out-of-sample properties that are crucial.

The conclusion is that the method used on nominal SP500 price data is very sensitive to the chosen start-period of data, just like the problem of estimation log-linear time-trend described above. The lack of stationarity in the VIBE-R series when used on nominal US stock-market data is the core of the problem.

SP500 - Real data

Deflating the stock prices with the consumer price index can possibly make better stationarity in the VIBE-R series, since real interest rates are known to be more stable than nominal interest rates.

Fig. 19



Source: Own excel graphics, based on case Shiller dataset

Linear time trend: lnP = 0,0016*t+4,5988+ e_t (t=1..N) SSE :sum(e^2)=346,6

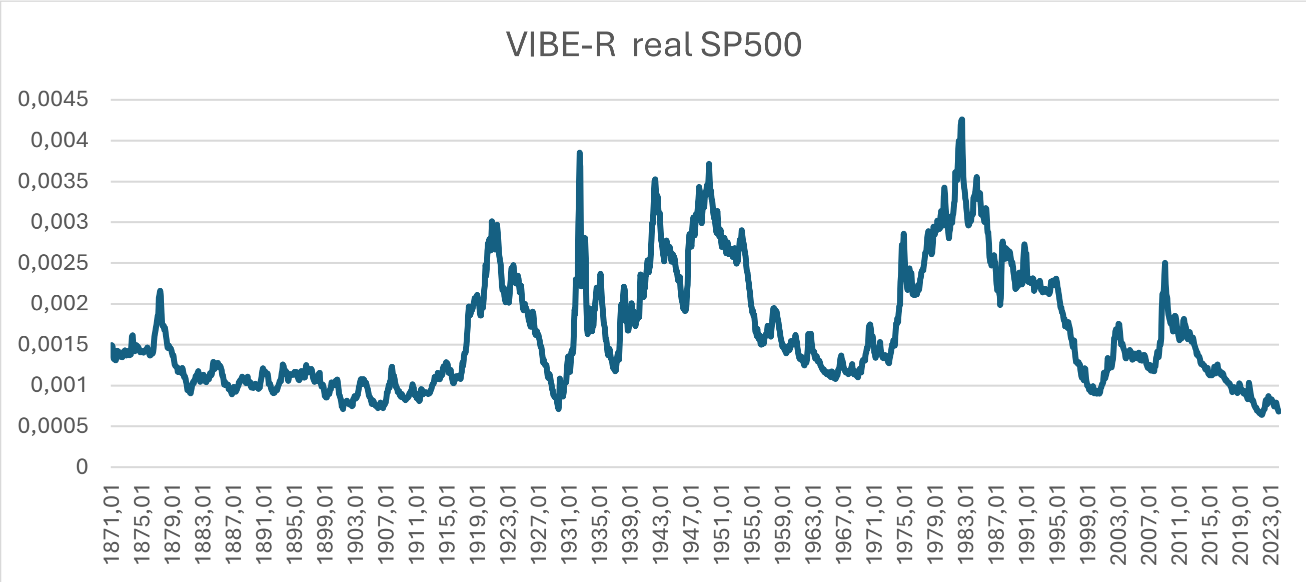
VIBE-path: $\ln P_t = \ln P_0 + \sum_{n=0}^{t-1} R_n + \ln R_0 - E(\ln R_t) + e_t$ (R_0= 0,00146728625240001)
SSE : sum(e^2) = 316,4

VIBE-path show lower SSE (P-value 0,0502)

There seems to be slightly better mean reversion to the trend in the real data than the nominal data above. The VIBE-path is based on the VIBE-R below in fig. 20, calibrated minimizing var($\ln R_t$):

$$\mathbf{R_t = EXP[\ Ln(R_{t-1}) + R_{t-1} - Ln(\frac{P_t}{P_{t-1}})\ ]} \quad (R_0 = 0{,}00146728625240001)$$

Fig 20



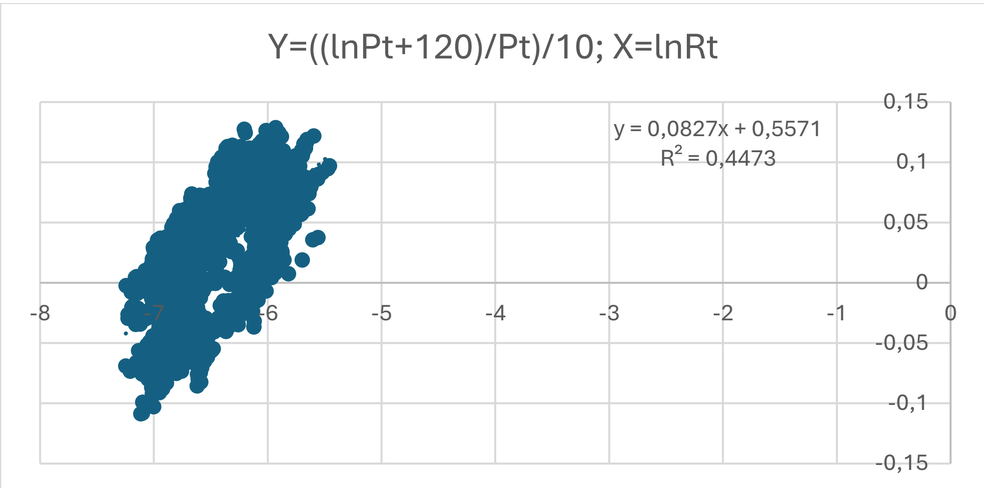
Source: Own calculations and excel graphics, based on case Shiller dataset

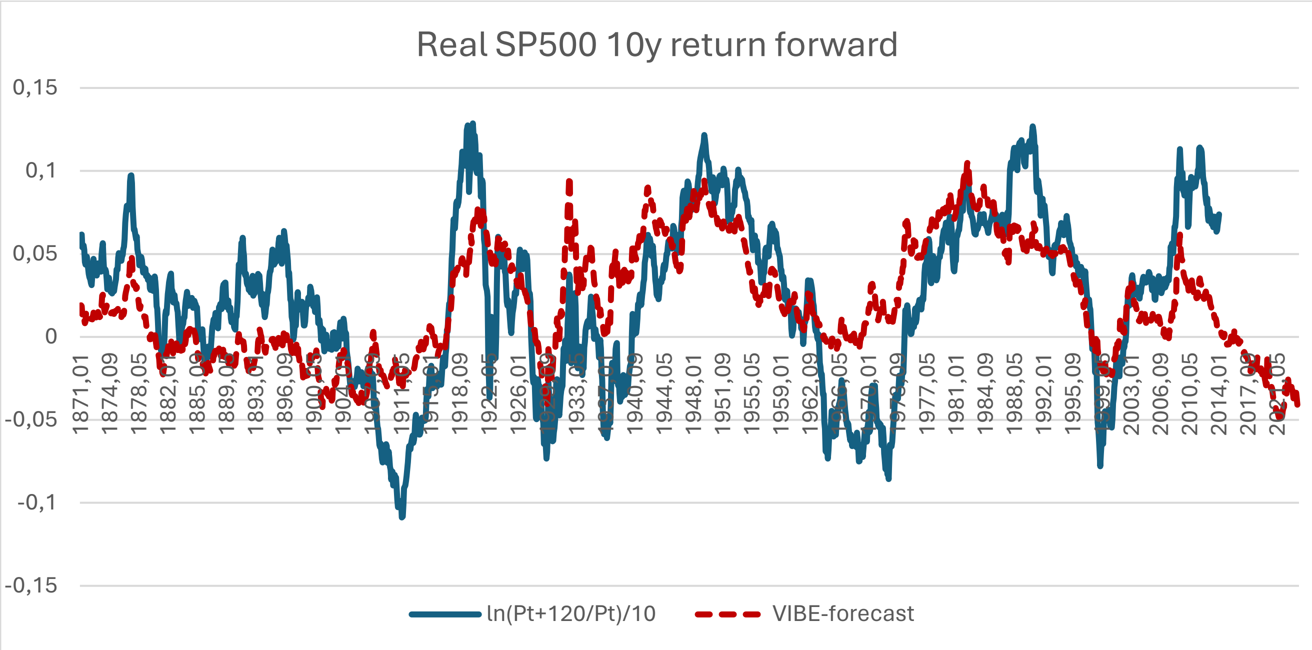
D.F. t-value: -1,647 H0: unit root: Cannot be rejected. Stationarity is not achieved.

The real R_t time series is not stationary although mean reversion occurs more often and standard deviation on real R_t (0,00072) is as expected lower than on the nominel R_t (0,00185).

The 10 years forecast model show a more modest correlation with real R_t:

Fig 21



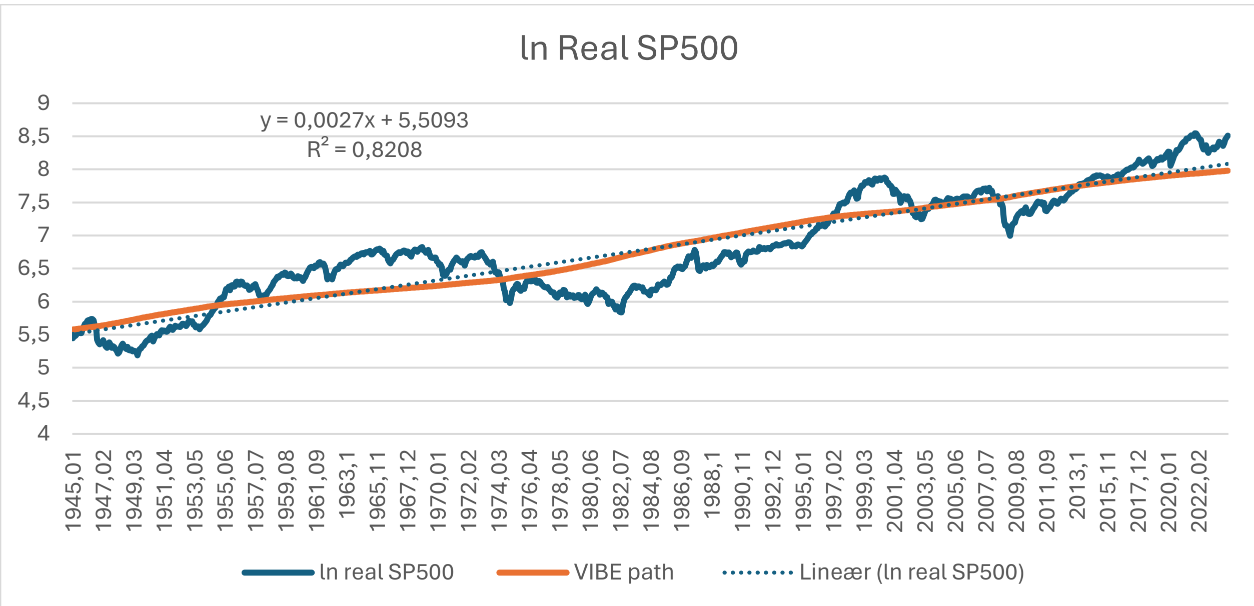


Source: Own calculations and excel graphics, based on case Shiller dataset

The with-in-sample are not impressive, but relatively stable changing the end-of-period. Out of sample, the real R_t model again predicts a horrible future for the real SP500 index with negative 10 years forward real return – very much like the nominal R_t model in fig. 15. But let us try to change the start period like we did with the nominal data.

Taking the same sub-period as the former analysis (1945-2024) :

Fig 22



Source: Own excel graphics, based on case Shiller dataset

Linear time trend: $\ln P = 0{,}0027*t+5{,}5093+ e_t$ (t=1..61) sum(e^2) = 114,7

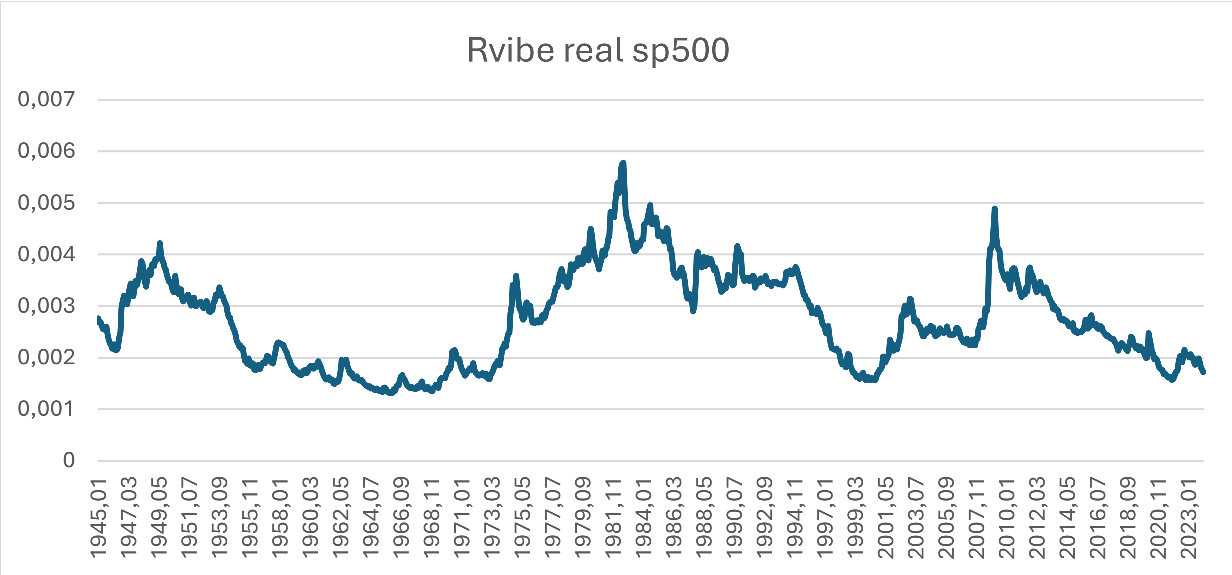
VIBE-path: $\ln P_t = \ln P_0 + \sum_{n=0}^{t-1} R_n + \ln R_0 - E(\ln R_t) + e_t$ (R_0= 0,00276087450978793) sum(e^2) = 111,96

Insignificant SSE difference.

The VIBE-path is based on the VIBE-R below in fig. 23, calibrated minimizing var($\ln R_t$):

$$\mathbf{R_t = EXP[\ Ln(R_{t-1})+R_{t-1}- Ln(\frac{P_t}{P_{t-1}})\ ]} \quad (R_0 = 0{,}00276087450978793)$$

Fig 23



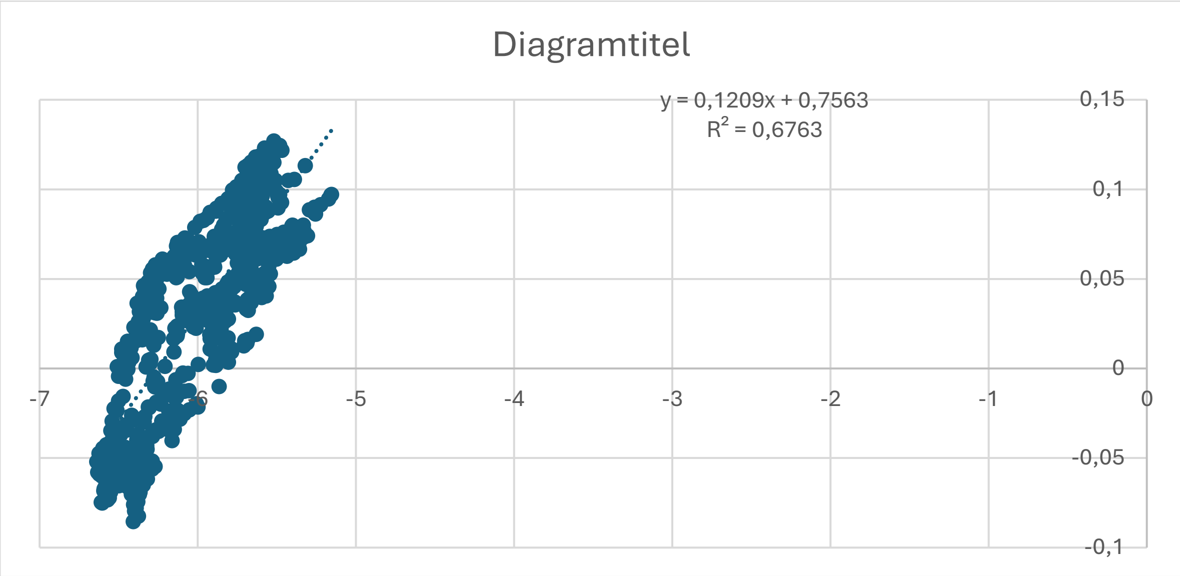
Source: Own calculations and excel graphics, based on case Shiller dataset

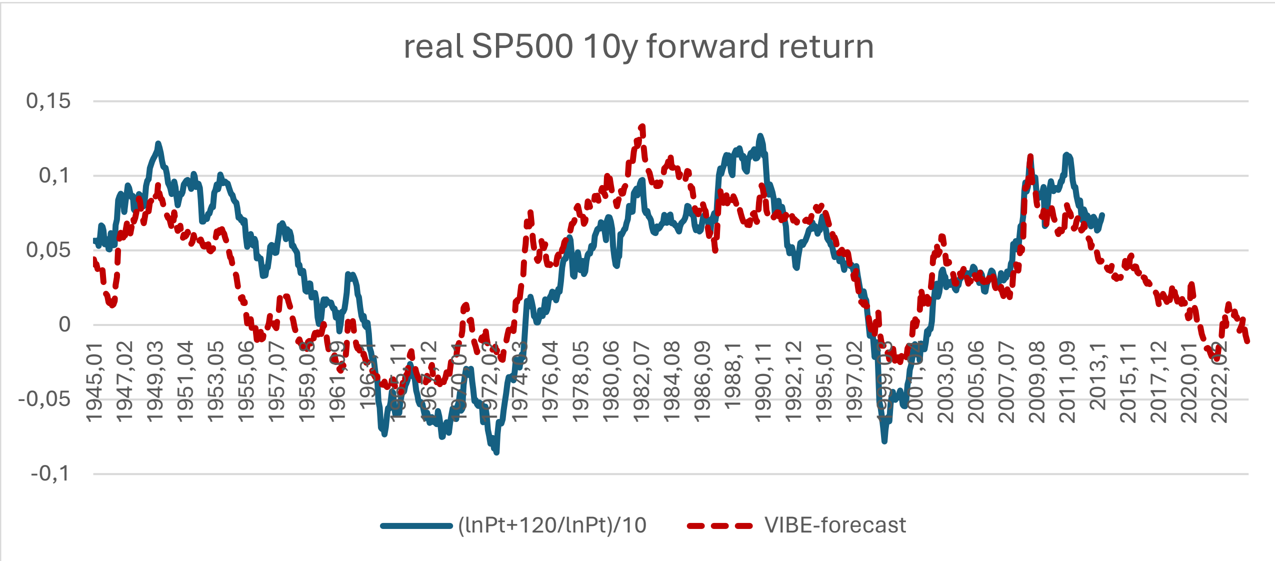
D.F. t-value: -1,647 H0: unit root : Cannot be rejected. Stationarity is not achieved.

Although the real R_t series is not stationary, the change of start-period did not impact the tail of the series very much.

The 10year model:

Fig 24





Source: Own calculations and excel graphics, based on case Shiller dataset

The sample period 1945-2024 also results in out of sample forecast predicting a very low (negative!) forward 10 year real return and a very high risk at the present stockprice, indicating a stock market bubble at the present prices.

However, the lack of stationarity is critical.

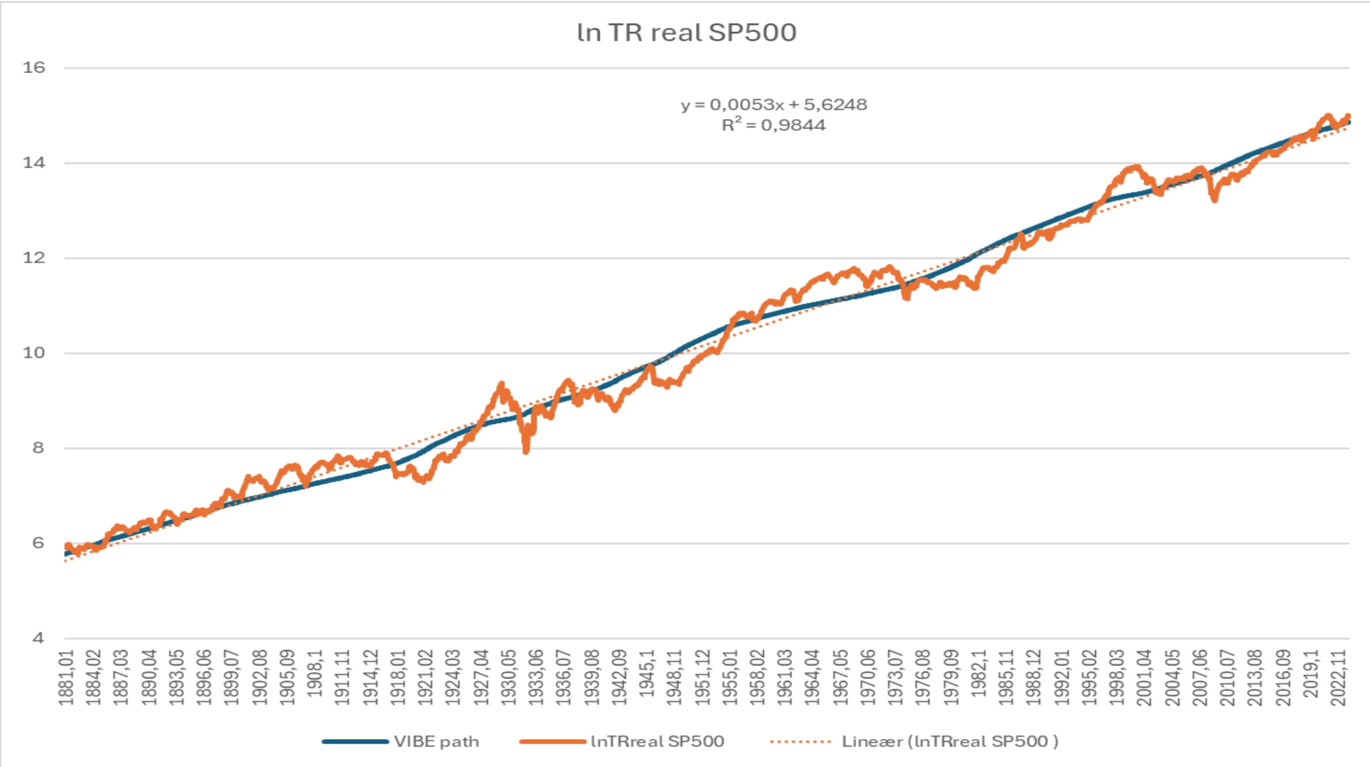
Sp500 real total return data:

Inflation is taken out of the real data above, but changes in pay-out policy could be another factor that prevents stationarity in R_t. A real total return stock price series could neutralize changes in pay-out policy over the data history, further improving the stationarity and mean-reversion to the trendline. Such a time series is also included in the Shiller dataset - To quote Shiller (Shiller 2024)

"As of September 2018, I now also include an alternative version of CAPE that is somewhat different. As documented in Bunn & Shiller (2014) and Jivraj and Shiller (2017), changes in corporate payout policy (i. e. share repurchases rather than dividends have now become a dominant approach in the United States for cash distribution to shareholders) may affect the level of the CAPE ratio through changing the growth rate of earnings per share. This subsequently may affect the average of the real earnings per share used in the CAPE ratio. ***A total return CAPE corrects for this bias through reinvesting dividends into the price index and appropriately scaling the earnings per share."***

Using Shillers real total return index below and later comparing VIBE-R with Shillers total return (TR) CAPE, some interesting results appears.

Fig 25



Source: Own excel graphics, based on case Shiller dataset

Linear time trend: lnP = 0,0053*t+5,6248+ e_t (t=1..61) SSE :sum(e^2) = 188,7

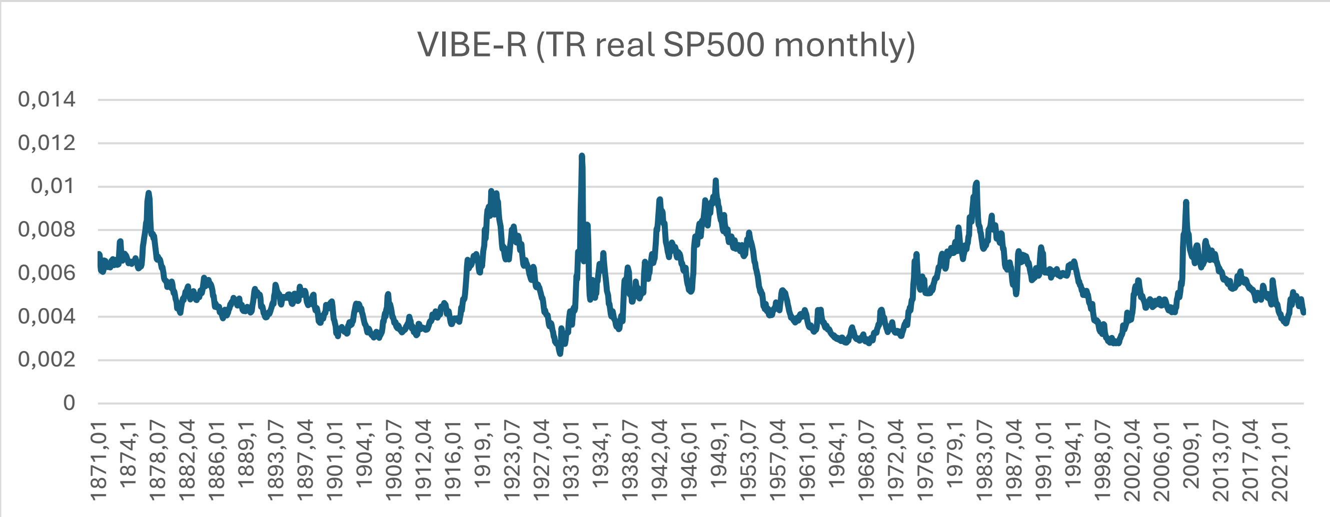
VIBE-path: $\ln P_t = \ln P_0 + \sum_{n=0}^{t-1} R_n + \ln R_0 - E(\ln R_t) + e_t$ (R_0=0,00676977578809851)
SSE : sum(e^2) = 158,8

VIBE-path show lower SSE (P-value 0,0502)

The VIBE-path is based on the VIBE-R below in fig. 26, calibrated minimizing var($\ln R_t$):

$$\mathbf{R_t = EXP[\, Ln(R_{t-1}) + R_{t-1} - Ln(\frac{P_t}{P_{t-1}})\,]}$$ (R_0 =0,00676977578809851)

Fig 26



Source: Own calculations and excel graphics, based on case Shiller dataset

D.F. t-value : -2,88 Ho: unit root: Can be rejected. Stationarity is achieved.

	1	2	3	4	5	6	7
ACF	0,99105	0,977463	0,963829	0,951072	0,937903	0,923266	0,907752
+/- crit	0,0457	0,0458	0,0458	0,0458	0,0458	0,0458	0,0458
PACF	0,99105	-0,2644	0,062	0,0264	-0,0577	-0,0698	-0,017
+/- 2*SE	0,0062	0,045	0,0466	0,047	0,047	0,047	0,047

The ACF and PACF indicates a ARÍMA (2,0,0) or ARIMA (3,0,0) model:

$lnR_t = 0{,}0567 + 1{,}2694 * lnR_{t-1} - 0{,}342 * lnR_{t-2} + 0{,}062 * lnR_{t-3} + e_t$

SE: (0,016) (0,0233) (0,0396) (0,0233)

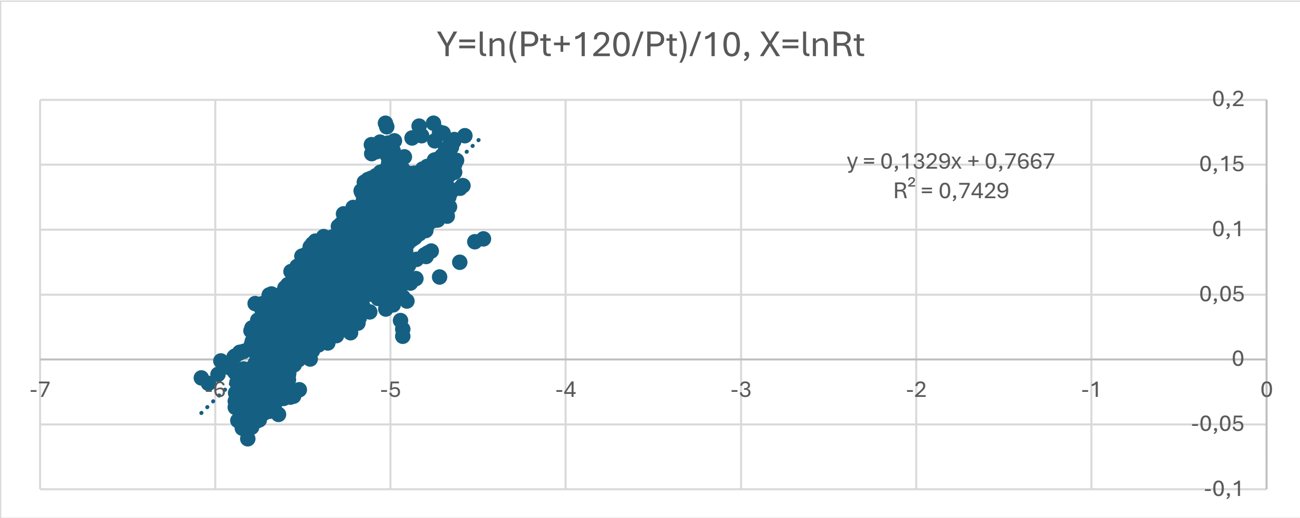
$e_t = 0{,}000003 - 0{,}0018 * e_{t-1}$

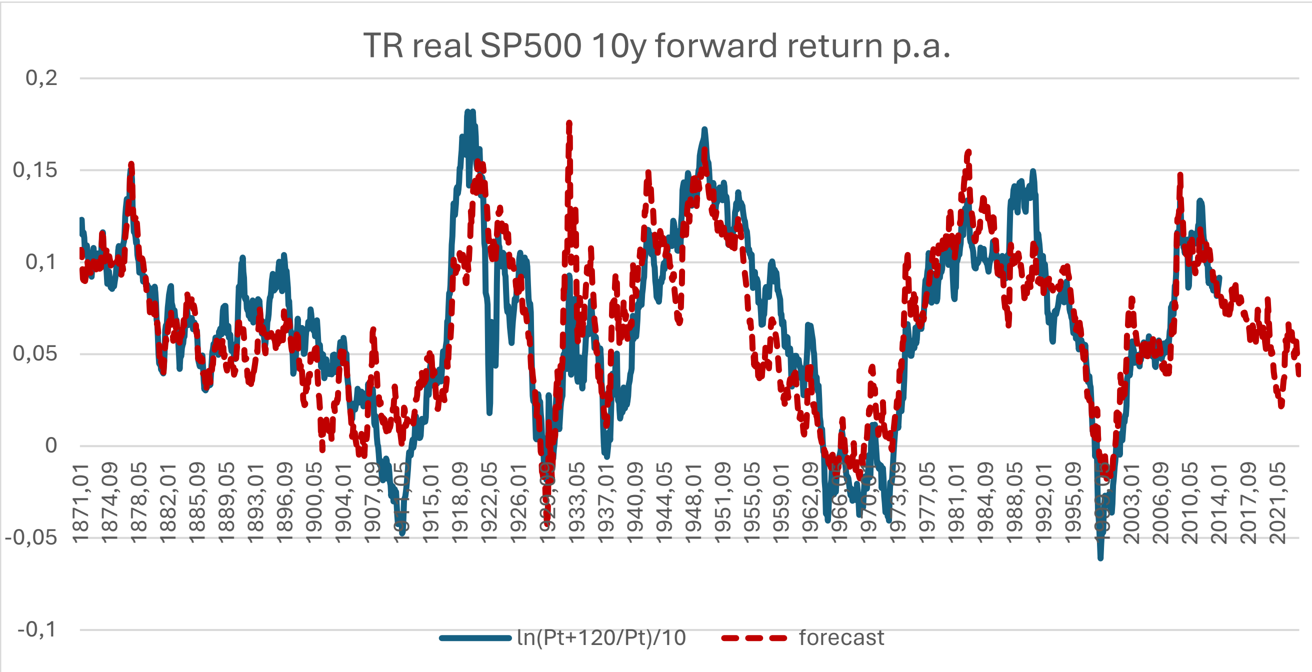
SE : (0,0009) (0,023)

No autocorrelation is detected in the error term e_t. (DW = 2,002)

The stationarity means, that out-of-sample results are much more robust to changes in the time of sample start as well as the end of sample. Estimating a 10y forecast model:

Fig 27





Source: Own calculations and excel graphics, based on case Shiller dataset

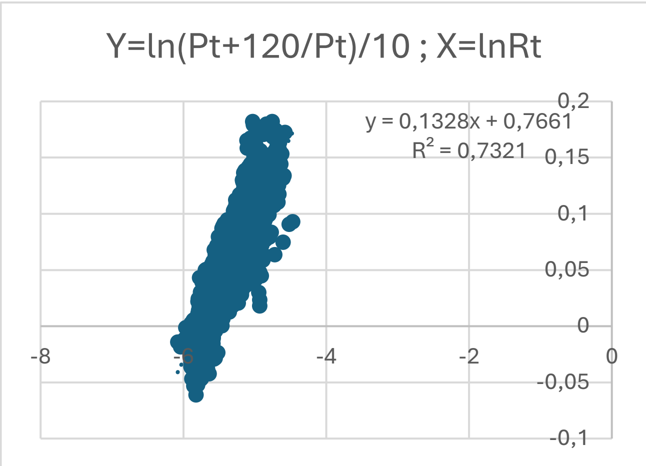
The within sample correlation is impressive. The out of sample forecast is much less dramatic than the results using nominel or real data. Only moderately lower 10 year forward return and moderately higher risk than historical average is predicted. The stationarity of the R_t series makes the out of sample forecast much more stable and robust when changing the sample period.

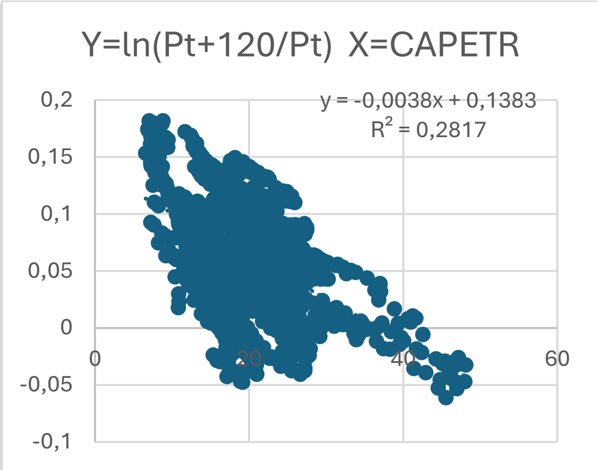
Comparing R_t with Shillers CAPE TR

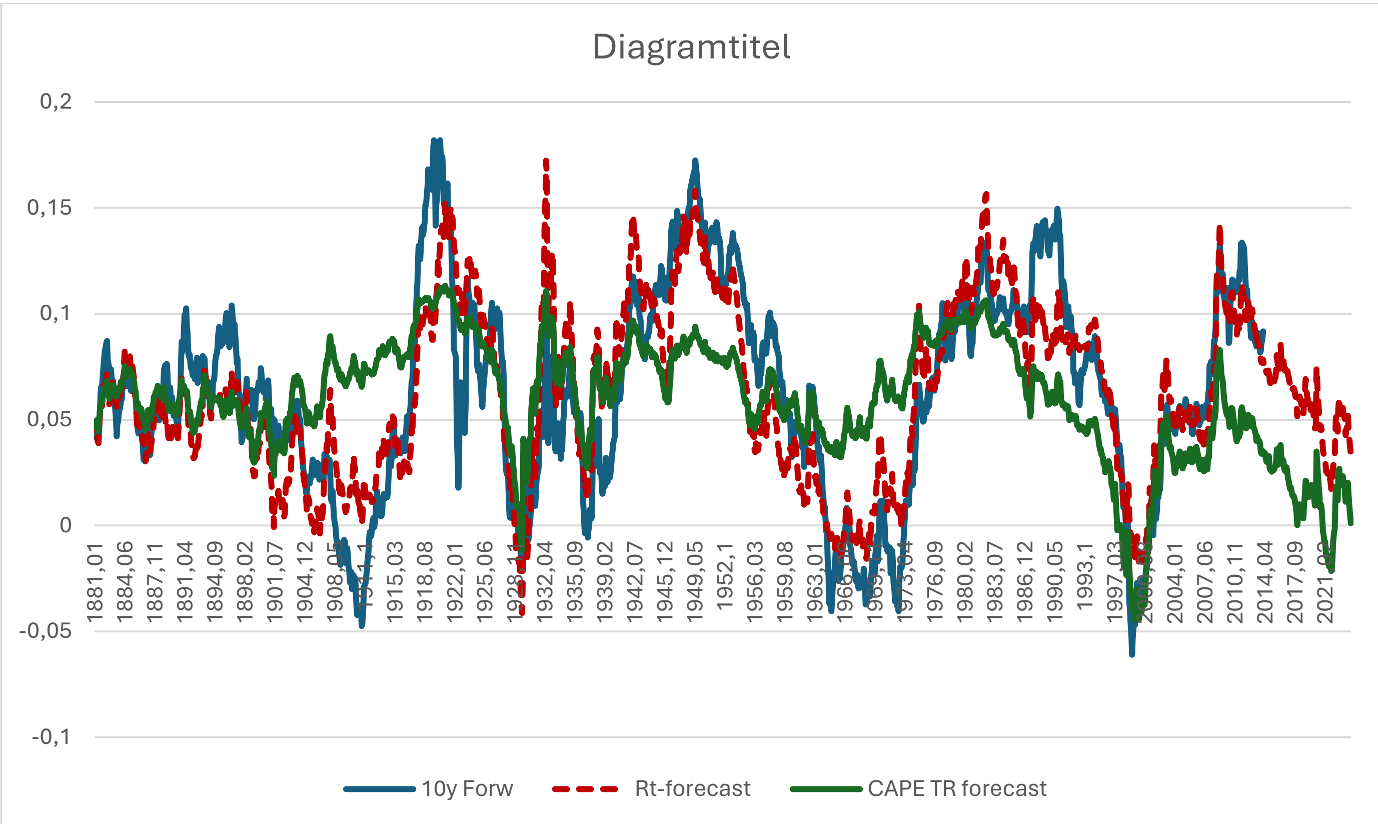
First let us compare the prediction power of VIBE-R compared to Shillers CAPE TR within sample: The CAPE TR series available is from 1881.01 to 2024.02.

(R_t-series is calibrated minimizing variance($\ln R_t$) starting 1881.01 with R_0= 0,00438749421551379)

Fig 28







Source: Own calculations and excel graphics, based on case Shiller dataset

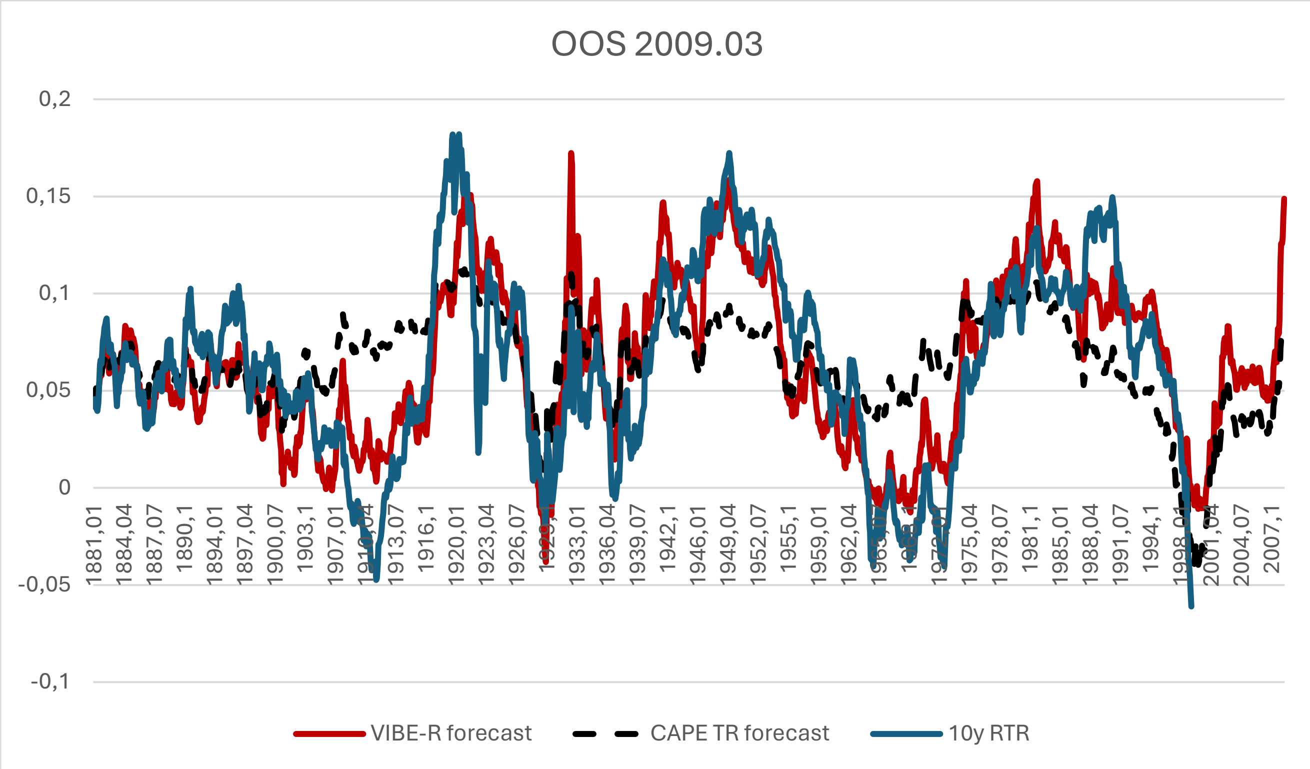
There is no doubt, that the R_t forecast outperforms CAPE RT forecast within sample. (P-value 0,000001) Out of sample CAPE TR predicts a lower return and higher risk than R_t.

Let us study some earlier out-of-sample examples – first let us look at the out of sample 2009:

Out of sample marts 2009.03:

(R_t-series is calibrated starting 1881.01 with R_0=0,00142533211660393)

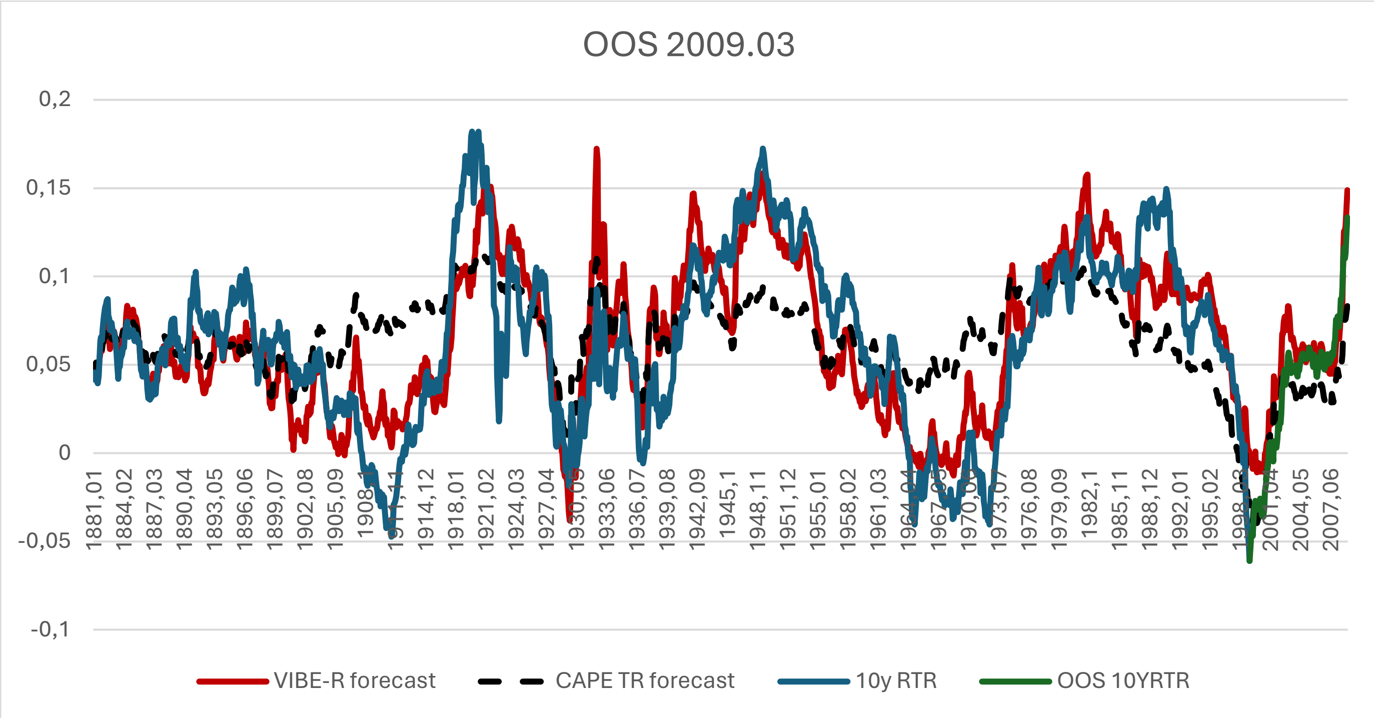
Fig 29



Source: Own calculations and excel graphics, based on case Shiller dataset

And the result 10 years after:

Fig 30



Source: Own calculations and excel graphics, based on case Shiller dataset

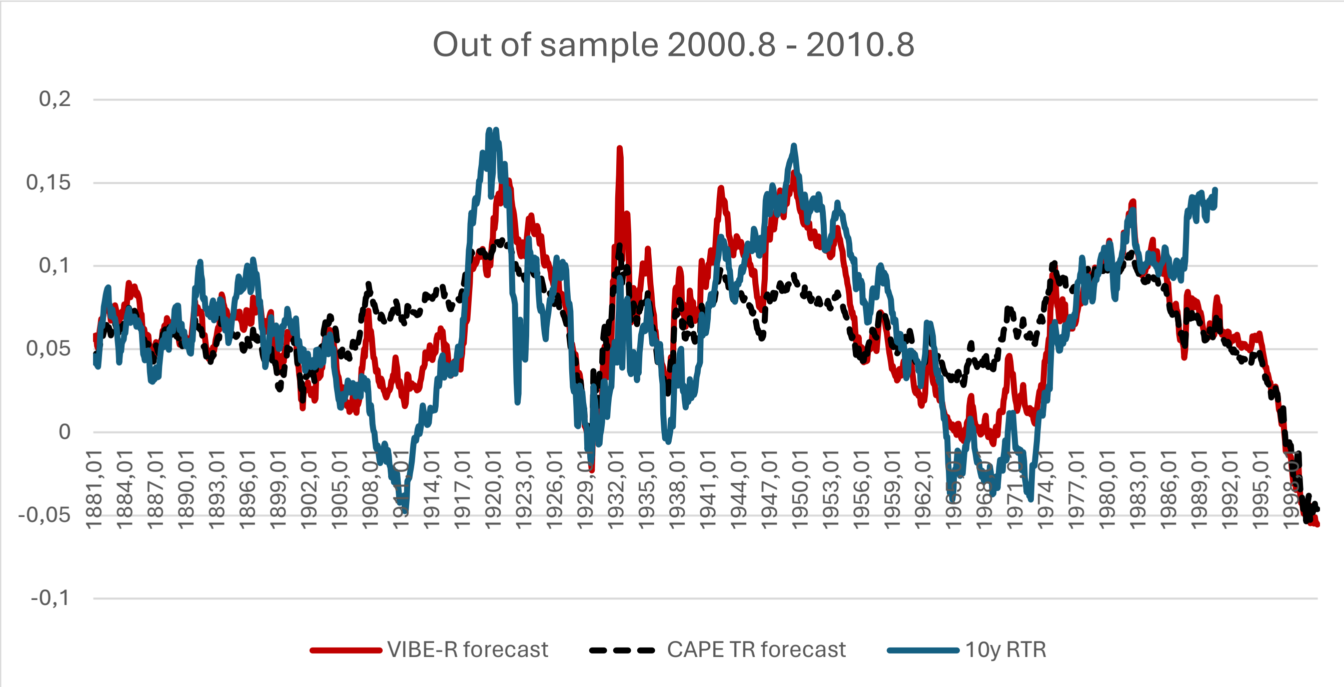
CAPE TR model : Within sample SSE :sum(e^2) = 2,47, out of sample SSE: sum(e^2)= 0,442

VIBE-R model: Within sample SSE: sum(e^2)= 0,97, out of sample SSE: sum(e^2) = 0,0405

VIBE-R shows significantly smaller SSE within sample (P-value: 0,000001) but insignificant smaller SSE out of sample

Out of sample august 2000.08:

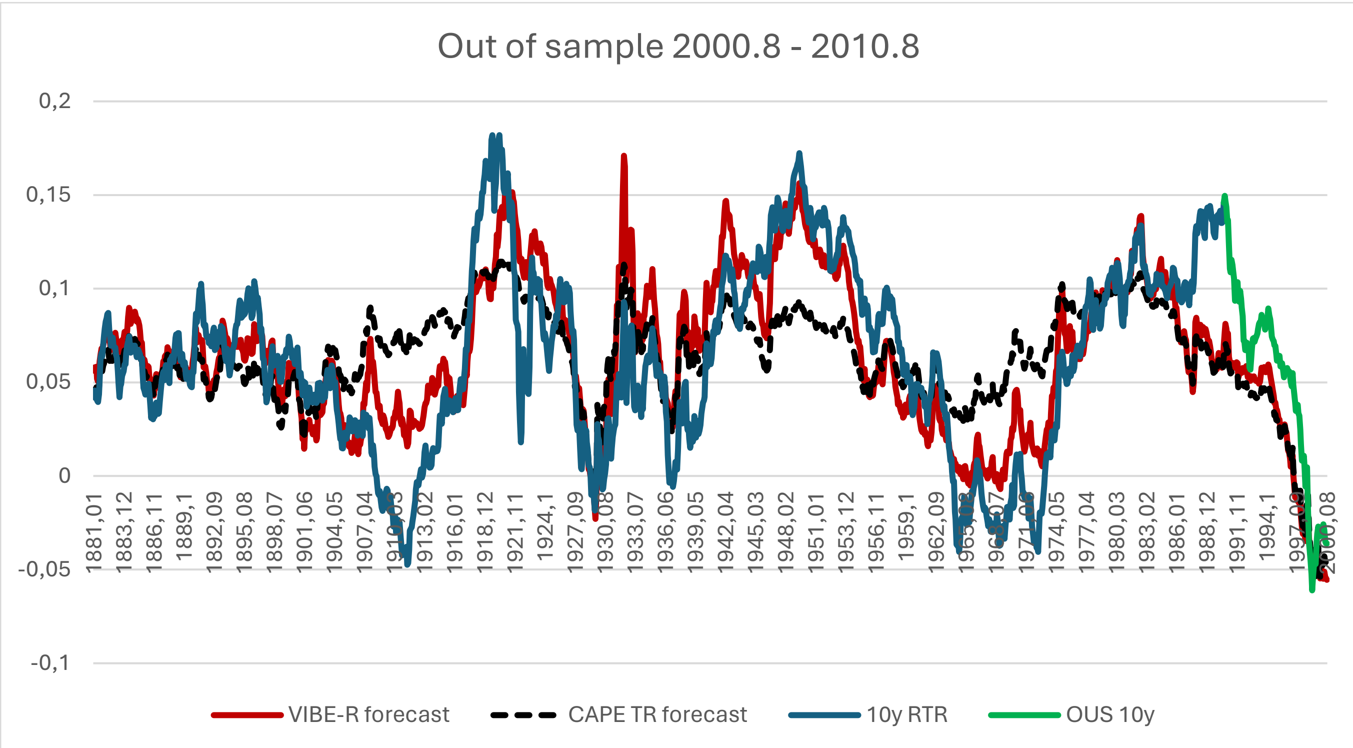
Fig 31



Source: Own calculations and excel graphics, based on case Shiller dataset

And the result 10 years after:

Fig 32



Source: Own calculations and excel graphics, based on case Shiller dataset

CAPE TR model: Within sample SSE : sum(e^2) = 2,28, out of sample SSE: sum(e^2)= 0,165

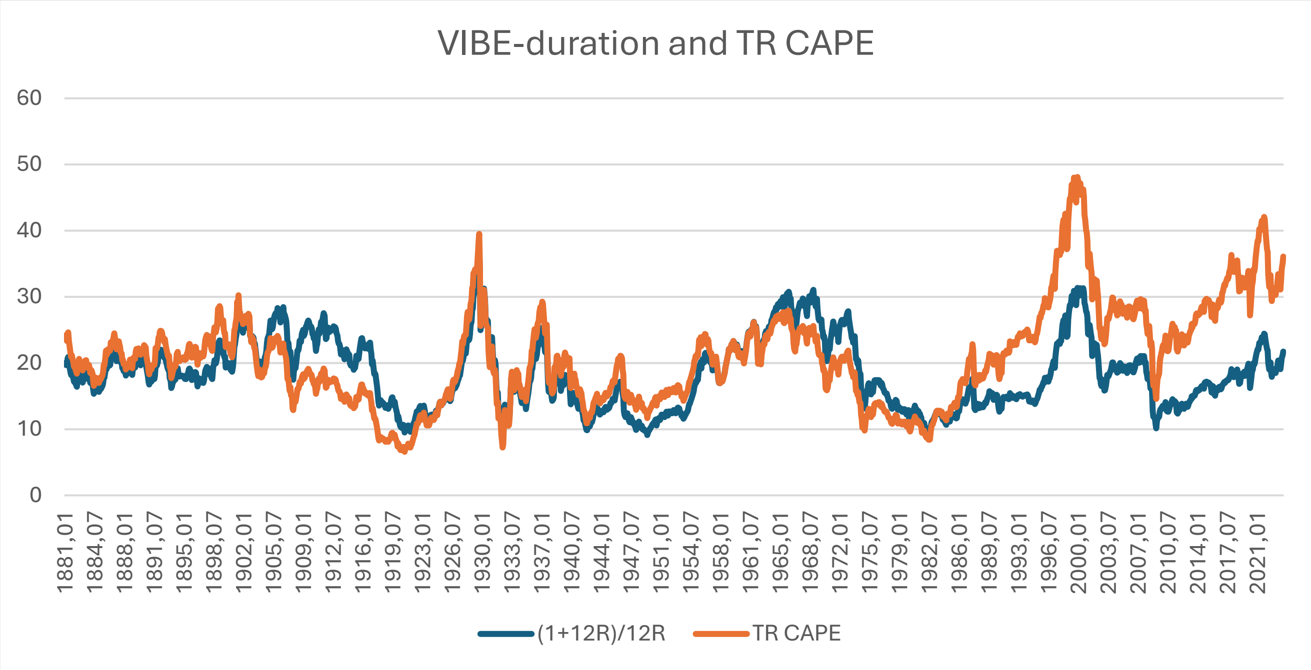
VIBE-R model: Within sample SSE: sum(e^2) = 1,087, out of sample SSE: sum(e^2) = 0,138

Within sample significant lower SSE in VIBE-R model (P-value 0,000001) Out of sample insignificant lower SSE in VIBE-R model.

How does VIBE-R duration compare to Shillers TR CAPE?

A direct comparison of VIBE-duration and TR CAPE shows remarkably similar properties within sample for the first 100 years – but the last 35 years they differ. TR CAPE predicts much higher present risk forward than R_t. However, the very high sensitivity of R_t to the crucial start parameter R_0 must be remembered. The jury is still out voting.

Fig 33



Discussions:

Why are long time series of asset prices so interesting to study?

Allow me a philosophical sidestep: Assume that the 12 Olympic Greek Gods - knowing the eternal future - were trading the SP500 stock index. There would be no uncertainty and therefore no volatility in the stock prices. There would be a smooth, continuously increasing – but not necessarily log-linear - trendline in prices, showing the true present value of stocks. The reason being, that the Gods would know how to profit from any deviation from the "devine olympic path" and trade those deviation away.

For us mere mortals, the future is not known, we do not know the “devine olympic path”. But fascinating to consider, that it does exist, we just do not know its exact location – and never will!

Prices on the stock market reflect our more or less wise expectations for the future, that are based on our ability to analyze the information available about the past and present. Continually, we discuss the degree of efficiency in the markets. But we can all agree that sometimes the future looks bright and prosperous, and optimism and greed thrive - sometimes the future looks bleak and dangerous, and anxiety and fear prevail. That is why the SP500 index does not follow a smooth continuously increasing trend but shows highs and lows and display more or less volatility.

Rational expectations about the future does not mean, that we can predict the future – we are most of the time wrong about the future - but rather, that we are not systematically wrong about the future. (Sargent, u.å.) Our “wrongness” is unsystematic but evenly distributed around the unknown “devine olympic path”.

This is all in good accordance with the random walk model and the corresponding log-linear time trend – except that the “divine olympic path” is not likely to be linear and the pro-cyclic nature of the random walk mentioned in the introduction. Historically – ex post – we can all acknowledge that the bull market results in higher risk and lower future return, and vice versa in a bear market.

This paper presents a new method to stationarize stochastic time series with an exponential trend. It is suggested that the VIBE-walk and VIBE-path model may be a better model than the random walk and loglinear trend model to extract the information contained in asset price data timeseries.

Applied on stock market data from Denmark and USA, the new method shows better prediction results compared with random walk and Shillers CAPE. Highly significant within sample, but only insignificantly better out of sample.

The VIBE-walk is not a stable stochastic process but an unstable chaos equation, highly dependent on the start parameter R_0. Like the random walk and loglinear trend model, the results out of sample also depend crucially on the time period chosen when using nominel data – but the results are less dependent on the period chosen when using real total return data, and the results are intriguing.

The crucial parameter R_0 has throughout the paper been estimated by minimizing the variance($\ln R_t$) within sample, stationarizing the R_t series. Other criteria have been tried but is left out of this paper, e.g.:

- Minimizing the SSE sum(e^2) in the forward return models or trend line deviation produce marginally better results within sample, but less good results out of sample.

- Calibrate the R_0 on nominal data, so the R_t series are tracking 10y government bond yields resulted in poor within sample results as well as out of sample results. Used on real data, the problem is that expected real bond yields are un-observable.

There are many other interesting implications and perspectives:

- The VIBE-walk method may be applied on real price data for exhaustible resources, calculating implicit real oil-yield, gold-yield, cobber-yield etc. encompassing and extending the simple Hotelling-rule (Dasgupta og Heal 1979)
- The VIBE-walk method may also be applied on nominal exchange rates, calculating implicit long bond interest differential, or implicit real house- and property yield.
- The method may be applied in portfolio management for better contra cyclical asset allocation between stocks, bonds and exhaustible resources.
- The Black Scholes universe for long options may be an interesting case. If the VIBE-duration can predict volatility medium to long term forward better than the random walk with drift or GARCH models (Engle 2001) , then the pricing of long options may be improved.

It is my hope that this paper could inspire students and scholars to investigate further and better into the subject than me. If it would lead to more stable markets, better asset allocation and better returns for long term investors in our future, it would be worth the effort.

Acknowledgement

A special thanks to research assistant Jacob Luis Nielsen for helping out in numerous ways. Also, valuable comment from Peter Løchte Jørgensen, University of Aarhus, Hans Jørgen Biede, Business Academy Aarhus and Jørgen Damgaard, Jyske Bank was much appreciated. They are of course in no way responsible for any errors and omissions in this working paper.

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Appendix 1

stdafv			0,02081			0,224261
middel			0,090736			-2,42767
		OMXC PI	VIBE-R	Duration	R-mean	ln VIBE-R
0	1964	9,87	0,062167	17,08578	-0,02857	-2,77794
1	1965	10,55	0,061886	17,15873	-0,02885	-2,78246
2	1966	10,21	0,068031	15,69907	-0,0227	-2,68778
3	1967	9,19	0,080912	13,3591	-0,00982	-2,51439
4	1968	10,21	0,078958	13,66498	-0,01178	-2,53884
5	1969	10,29	0,084739	12,80097	-0,006	-2,46818
6	1970	9,27	0,102386	10,76691	0,01165	-2,279
7	1971	9,02	0,116635	9,57375	0,025899	-2,14871
8	1972	17,12	0,069018	15,48904	-0,02172	-2,67339
9	1973	17,12	0,073949	14,52276	-0,01679	-2,60437
10	1974	13,48	0,101145	10,88675	0,010409	-2,2912
11	1975	18,03	0,083651	12,95446	-0,00709	-2,4811
12	1976	18,03	0,090949	11,99515	0,000213	-2,39745
13	1977	17,85	0,100625	10,93787	0,009889	-2,29635
14	1978	16,76	0,118535	9,436337	0,027798	-2,13255
15	1979	15,66	0,142763	8,004635	0,052026	-1,94657
16	1980	17,58	0,146753	7,814176	0,056016	-1,91901
17	1981	24,32	0,122848	9,140167	0,032111	-2,09681
18	1982	27,32	0,123626	9,088934	0,032889	-2,0905
19	1983	58,47	0,065371	16,29731	-0,02537	-2,72768
20	1984	45,63	0,089428	12,18221	-0,00131	-2,41432
21	1985	65,30	0,068333	15,63427	-0,0224	-2,68337
22	1986	52,73	0,090604	12,03707	-0,00013	-2,40126
23	1987	49,73	0,105192	10,50647	0,014455	-2,25197
24	1988	74,32	0,078193	13,78888	-0,01254	-2,54858
25	1989	99,18	0,063356	16,7838	-0,02738	-2,75898
26	1990	86,07	0,077786	13,85584	-0,01295	-2,5538
27	1991	96,45	0,075027	14,32854	-0,01571	-2,58991
28	1992	71,58	0,108962	10,17752	0,018225	-2,21676
29	1993	100,00	0,086979	12,49698	-0,00376	-2,44208
30	1994	95,36	0,099505	11,04971	0,008769	-2,30754
31	1995	100,00	0,104811	10,54101	0,014074	-2,2556
32	1996	130,01	0,089526	12,16998	-0,00121	-2,41323
33	1997	184,47	0,069005	15,49175	-0,02173	-2,67358
34	1998	175,41	0,077753	13,86119	-0,01298	-2,55421
35	1999	213,86	0,068931	15,50736	-0,02181	-2,67466
36	2000	247,62	0,063781	16,67864	-0,02696	-2,7523
37	2001	211,86	0,079456	13,58554	-0,01128	-2,53255
38	2002	166,56	0,109424	10,13874	0,018688	-2,21252
39	2003	216,59	0,093879	11,65201	0,003143	-2,36575
40	2004	263,32	0,084819	12,78979	-0,00592	-2,46723

	41	2005	367,50	0,066154	16,11621	-0,02458	-2,71577
	42	2006	423,43	0,061343	17,30185	-0,02939	-2,79128
	43	2007	446,69	0,061827	17,17412	-0,02891	-2,78341
	44	2008	227,98	0,128866	8,75997	0,03813	-2,04898
	45	2009	301,26	0,110934	10,01439	0,020197	-2,19882
	46	2010	395,20	0,094485	11,58369	0,003749	-2,35931
	47	2011	325,19	0,126205	8,923594	0,035469	-2,06984
	48	2012	404,00	0,115251	9,676731	0,024514	-2,16064
	49	2013	517,00	0,101062	10,89493	0,010325	-2,29202
	50	2014	608,00	0,095075	11,51804	0,004338	-2,35309
	51	2015	786,00	0,080879	13,36413	-0,00986	-2,5148
	52	2016	723,00	0,095334	11,48948	0,004597	-2,35037
	53	2017	836,00	0,090695	12,02603	-4,2E-05	-2,40026
	54	2018	752,00	0,110397	10,05821	0,019661	-2,20367
	55	2019	946,00	0,098001	11,20401	0,007264	-2,32278
	56	2020	1224,00	0,083541	12,97016	-0,0072	-2,48242
	57	2021	1480,00	0,075111	14,3137	-0,01563	-2,58879
	58	2022	1392,00	0,086088	12,61599	-0,00465	-2,45238
	59	2023	1569,00	0,083243	13,01302	-0,00749	-2,48599
04/03/2024	60	2024	1890,00	0,075104	14,31494	-0,01563	-2,58889